Sr. No. 081

ENTRANCE TEST-2024

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

MATH FOR STATISTICS

Question Booklet Series **A**

Total Questions : 60
Time Allowed : 70 Minutes

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be **'Negative Marking'** for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
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SEAL

1. Given that $f(x) = (x^2 - 4)(x - 1)^2$ and $g(x) = x^2$ and $h(x) = \frac{f'(x)}{g'(x)}$, then for what value of k, $\lim_{x \to 1} h(x) = k$?

(A) 1

(B) 0

(C) –1

(D) 4

2. The value of $\lim_{x \to 0} \frac{\sin(x^3)}{x^2 \sin x}$ is equal to :

(A) ∞

(B) 0

(C) 1

(D) Can't say

3. For the function $f(x) = -x + \frac{1}{x^2}$; for what value of k; $f'(-1) + f'(-\frac{1}{2}) = 2^k$?

(A) 4

(B) – 4

(C) 1

(D) 0

4. If $f(x) = a^x$, then $f^n(0)$ is equal to :

(A) 0

(B) 1

(C) $\frac{(\log a)^n}{n}$

(D) None of these

5. The angle between the radius vector and the tangent is given by :

(A) $\tan^{-1} \phi = r\frac{d\theta}{dr}$

(B) $\tan \phi = r\frac{d\theta}{dr}$

(C) $\tan \phi = \frac{r}{\theta}\frac{d\theta}{dr}$

(D) $\tan \phi = r\frac{dr}{d\theta}$

6. The curvature of the circle of radius α is :

(A) α

(B) $\frac{1}{\alpha^2}$

(C) $\frac{1}{\alpha}$

(D) α^2

7. A double point on a curve is said to be cusp, if two tangents of the curve are :

(A) real and distinct

(B) imaginary and distinct

(C) real and coincident

(D) none of these

8. The asymptotes to the hyperbola $9x^2 - 16y^2 = 144$ are given by :

(A) $y = \pm\frac{3}{4}x$

(B) $y = \pm\frac{5}{4}x$

(C) $y = \pm\frac{7}{4}x$

(D) None of these

9. For what value of α and β, the following is a Maclaurin's series?

$$a^x = 1 - \alpha \log a + \frac{x^2}{2}(\log a)^2 - \beta \frac{x^3}{6}(\log a)^3 + ...$$

(A) $\alpha = 1, \beta = 1$

(B) $\alpha = x, \beta = 1$

(C) $\alpha = -1, \beta = -1$

(D) $\alpha = -x, \beta = -1$

10. If f and g are two differentiable functions in (0,1) satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in (0,1)$, the following holds:

(A) $2f'(c) = g'(c)$

(B) $2f'(c) = 3g'(c)$

(C) $f'(c) = g'(c)$

(D) $f'(c) = 2g'(c)$

11. Given $f(0) = 4$ and $f'(x) = \frac{3}{x^2 + 2}$, the lower bound of f(2) estimated by Mean value theorem is:

(A) 0

(B) 5

(C) 7

(D) 12

12. The function f(x) with $f'(k) = 0$ attains its maximum value at $x = k$ if:

(A) $f''(k) = 0$

(B) $f''(k) > 0$

(C) $f''(k) < 0$

(D) None of these

13. Using method of partial fractions while integrating $\frac{x}{(x-1)(x^2+4)} = \frac{A}{(x-1)} + \frac{Bx + C}{x^2 + 4}$, the value of B is:

(A) $-\frac{1}{5}$

(B) $\frac{1}{5}$

(C) $\frac{2}{5}$

(D) $\frac{4}{5}$

14. The value of $\int \cos^2 x \, dx = ?$

(A) $\frac{x}{2} + \frac{\sin 2x}{2} + c$

(B) $\frac{x}{2} + \frac{\sin 2x}{4} + c$

(C) $\frac{x}{2} - \frac{\sin 2x}{4} + c$

(D) $\frac{x}{2} + \frac{\cos 2x}{4} + c$

15. The value of the integral $\int_0^2 \frac{dx}{x^2 + 4} = ?$

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{8}$

(D) None of these

16. The value of $\int \frac{dx}{e^x - 1} = ?$

(A) $\log\left|\frac{e^x - 1}{e^x}\right| + c$

(B) $\log\left|\frac{e^x + 1}{e^x}\right| + c$

(C) $-\log\left|\frac{e^x - 1}{e^x}\right| + c$

(D) None of these

17. The integrating factor for the differential equation $\frac{dy}{dx} + 2xy = 2e^{-x^2}$ is :

(A) x^2

(B) e^{x^2}

(C) e^{-x^2}

(D) None of these

18. Which of the following is Bernoulli's type differential equation ?

(A) $\frac{dx}{dy} + Px = Qx^n$

(B) $\frac{dx}{dy} + Py = Qy^n$

(C) $\frac{dx}{dy} - \frac{P}{x} = Qy^n$

(D) All of them

19. The solution of $(D^3 + 6D^2 + 11D + 6)y = 0$ is :

(A) $y = c_1e^x + c_2e^{2x} + c_3e^{3x}$

(B) $y = c_1e^{-x} + c_2e^{-2x} + c_3e^{-3x}$

(C) $y = (c_1 + c_2x)e^x + c_3e^{2x}$

(D) $y = (c_1 + c_2x + c_3x^2)e^{6x}$

20. The value of $\int_0^a \varphi(x)dx = ?$

(A) $-\int_0^a \varphi(a - x)dx$

(B) $\int_0^a \varphi(2x + a)dx$

(C) $-\int_0^a \varphi(a + x)dx$

(D) $\int_0^a \varphi(a - x)dx$

21. The miscellaneous forms of differential equations mainly deals with :

(A) first order and higher degrees

(B) first degree and higher orders

(C) first order and first degree

(D) none of these

22. The solution of $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx} = 2$ is given by :

(A) $(y - x - a)(y - 2x - a)$

(B) $(y + 2x - a)(y - x - a)$

(C) $(y + x - a)(y + 2x - a)$

(D) $(y + x - a)(y - 2x - a)$

23. Which of the following is the complete primitive of $y = px + 2\sqrt{1+p^2}$?

(A) $\frac{(y-cx)^2}{2} = 1 + c^2$

(B) $\frac{(y-cx)^2}{4} = c^2$

(C) $\frac{(y-cx)}{4} = 1 + c^2$

(D) $\frac{(y-cx)^2}{4} = 1 + c^2$

24. The solution of $y = 3xp + 6y^2p^2$ by using $u = y^3$ is given by :

(A) $y^3 = cx + \frac{1}{3}c^2$

(B) $y^3 = cx + \frac{2}{3}c^2$

(C) $y^3 = x - \frac{1}{3}c^2$

(D) $y^3 = cx - \frac{2}{3}c^2$

25. The value of $J^2_{-\frac{1}{2}}(x) + J^2_{\frac{1}{2}}(x) = ?$

(A) $\frac{2}{\pi x}$

(B) $\sqrt{\frac{2}{\pi x}}$

(C) 1

(D) None of these

26. The value of k, for which $J'_0(x) = k$?

(A) $J_{-1}(x)$

(B) $-J_1(x)$

(C) 0

(D) None of these

27. Which of the following is a Legendre differential equation of order n ?

(A) $x^2y_2 + xy + (x^2 - n^2)y = 0$

(B) $(1-x^2)y_2 - 2xy_1 + n(n+1)y = 0$

(C) $y_2 - 2xy_1 + n(n+1)y = 0$

(D) $(1+x^2)y_2 + 2xy_1 + n(n+1)y = 0$

28. The Wronskian W (sin x, sin x – cos x) is :

(A) 0

(B) –1

(C) 1

(D) None of these

29. The partial differential equation formed by eliminating arbitrary constants a and b from $z = ax + by + ab$, is given by :

(A) $z = px + qy$

(B) $z = px + qy + pq$

(C) $z = \frac{x}{p} + \frac{y}{q} + \frac{1}{pq}$

(D) None of these

30. The complete solution of the partial differential equation $q = 3p^2$ is given by :

(A) $z = ax + 3a^2y + c$

(B) $z = 3a^2x + ay + c$

(C) $z = ax + (1-3a)y + c$

(D) None of these

31. The partial differential equation

$\frac{\partial^2 z}{\partial x^2} + 3x\frac{\partial^2 z}{\partial x \partial y} - y\frac{\partial^2 z}{\partial y^2} = 0$ is parabolic if :

(A) $9x^2 = 4y$

(B) $9x^2 + 4y = 0$

(C) $9x^2 + 4y > 0$

(D) $9x^2 + 4y < 0$

32. For the partial differential equation $p^2 + q^2 = 1$, the singular solution :

(A) exists and is given by $x^2 + y^2 = 1$

(B) exists and is given by $x^2 - y^2 = 1$

(C) exists and is given by $x^2 = y^2$

(D) does not exist

33. Choose the incorrect statement from the following :

(A) The Set A is denumerable if it is equivalent to the set of positive integers N

(B) Every infinite set has a denumerable subset

(C) Every infinite set is equivalent to one of its proper subsets

(D) None of them

34. The interval [0,1] is :

(A) countable

(B) uncountable

(C) proper subset of cantors set

(D) none of these

35. Which of the following is a consequence of Archimedean property in R ?

(A) For every real number x, $\exists$ an integer $n > 0$, such that $n > x$

(B) For every real number x, $\exists$ an integer m such that $m < x$

(C) For every positive real number x, $\exists$ a natural number n such that $\frac{1}{n} < x$

(D) All of them

36. Which of the following is not a field ?

(A) set of rational numbers

(B) set of real numbers

(C) set of irrational numbers

(D) set of complex numbers

37. For which of the sequence limit does not exist ?

(A) $\{(-1)^n\}$

(B) $\left\{n - \frac{1}{n}\right\}$

(C) $\{n\}$

(D) All of them

38. Which of the following sequence does not oscillate ?

(A) $\left\{2-\frac{1}{2^{n-1}}\right\}$

(B) $\{\cos n\pi\}$

(C) $\left\{\sin\frac{n\pi}{2}\right\}$

(D) $\left\{\frac{(-1)^n}{5}\right\}$

39. Every bounded sequence of real numbers has :

(A) a finite subsequence

(B) divergent subsequence

(C) convergent subsequence

(D) none of these

40. The value of $\lim_{n\to\infty}\left(1+\frac{k}{n}\right)^n = ?$ is :

(A) 0

(B) 1

(C) e^n

(D) e^k

41. Find the value of k for which $\sum_{n=1}^{\infty}\frac{1}{n^2} = k$:

(A) 0

(B) 1

(C) $\frac{\pi^2}{6}$

(D) k does not exist

42. Which of the following is a divergent series ?

(A) $\sum\frac{1}{\sqrt{n}}$

(B) $\sum\frac{1}{2^n}$

(C) $\sum\frac{1}{n!}$

(D) $\sum_{n=1}^{\infty}\frac{1}{n^2}$

43. The series $\sum_{n=0}^{\infty}\frac{n^{1-3n}}{4^{2n}}$:

(A) oscillates finitely

(B) oscillates infinitely

(C) diverges

(D) converges

44. The radius of convergence of the geometric series $1+x+x^2+x^3+\ldots\ldots$ is :

(A) 0

(B) 1

(C) 2

(D) 3

45. If the radius of convergence of $\sum a_n x^n$ is 5, then the radius of convergence of $\sum a_n(x+2)^n$ is :

(A) –2

(B) 2

(C) 1

(D) 3

46. Let k be a positive integer, the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} z^n$ is :

(A) k

(B) k^k

(C) k^{-k}

(D) ∞

47. Let R denotes the radius of converges of power series $\sum_{k=1}^{\infty} kx^n$, then

(A) $R > 0$ and the series is convergent on $[-R, R]$

(B) $R > 0$ and the series converges at $x = -R$ but does not converge at $x = R$

(C) $R > 0$ and the series does not converge outside $(-R, R)$

(D) $R = 0$

48. The product of two odd permutations is :

(A) odd

(B) even

(C) even or odd

(D) neither even nor odd

49. For a group G, if $a, b \in G$ such that $(ab)^{-1} = a^{-1}b^{-1}$, then G is a/an :

(A) commutative semigroup

(B) abelian group

(C) non-abelian group

(D) none of these

50. The number of odd permutations in a symmetric group S_4 is :

(A) 2

(B) 4

(C) 8

(D) 12

51. The order of w^2 in a group $G = \{1, w, w^2\}$; $\left(\text{where } w = \frac{-1+i\sqrt{3}}{2}\right)$ is :

(A) 1

(B) 2

(C) 3

(D) None of these

52. If C(G) and N (a) respectively denote the centre of a group G and normalizer of an element $a \in G$, then which of the following is correct ?

(A) $N(a) = C(G)$

(B) $N(a) \subseteq C(G)$

(C) $C(G) \subseteq N(a)$

(D) None of these

53. Choose the incorrect statement from the following :

(A) Every homomorphic image of a group is isomorphic to some quotient group

(B) The kernel of a homomorphism is a normal subgroup

(C) Intersection of two subgroups is always a subgroup

(D) None of them

54. Choose the incorrect statement from the following :

(A) If the index of a normal subgroup in a group is 2, then it is normal

(B) Intersection of two normal subgroups is a normal subgroup

(C) Every subgroup of an abelian group is normal

(D) None of these

55. The number of proper subgroups in a group of order 19 is :

(A) 18

(B) 9

(C) 2

(D) zero

56. The non-abelian group exists for which of the following first odd integer ?

(A) 21

(B) 23

(C) 27

(D) 33

57. The minimum number of elements required in a ring to qualify for a field is :

(A) 1

(B) 2

(C) 3

(D) any finite number

58. The ideal (2) in the ring Z of integers is :

(A) maximal

(B) prime

(C) principal

(D) all of these

59. An integral domain comprising of _____ elements is a field.

(A) two

(B) three

(C) finite

(D) infinite

60. Which of the following is a zero divisor of an elements $\overline{2}$ in a ring $<Z6, +, .>$?

(A) $\overline{3}$

(B) $\overline{2}$

(C) $\overline{0}$

(D) $\overline{4}$

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SEAL

1. Second quartile is also known as :

(A) median

(B) mode

(C) mean

(D) geometric mean

2. For a symmetric distribution, the first and third quartiles are respectively 27 and 49. The value of the median is :

(A) 30

(B) 35

(C) 38

(D) 45

3. For comparing the variability of two series which are in different units, which of the following measures is used ?

(A) standard deviation

(B) coefficient of variation

(C) mean deviation from mean

(D) inter-quartile range

4. The arithmetic mean of numbers 0, 1, 2, 3, ..., n is :

(A) $\frac{n(n+1)}{2}$

(B) $\frac{n}{2}$

(C) $\frac{n(n+1)^2}{2}$

(D) $\frac{(n+1)}{2}$

5. Correlation coefficient is independent of change of :

(A) origin

(B) scale

(C) neither (A) nor (B)

(D) both (A) and (B)

6. When the correlation coefficient is zero, the two lines of regression are :

(A) coincident

(B) parallel

(C) perpendicular

(D) none of these

7. If $\sigma_x^2 = 4\sigma_y^2$ and r = 0.25, then the regression coefficient b_{xy} is :

(A) 1.0

(B) 0.7

(C) 0.6

(D) 0.5

8. The rank correlation coefficient is given by :

(A) $1+\frac{6\sum D^2}{n^3-n}$

(B) $1-\frac{6\sum D^2}{n^3+n}$

(C) $1-\frac{6\sum D^2}{n^3-n}$

(D) $\frac{\sum(X-\overline{X})(Y-\overline{Y})}{\sqrt{\sum(X-\overline{X})^2}\sqrt{\sum(Y-\overline{Y})^2}}$

9. A set of all possible outcomes of a statistical experiment is called :

(A) sample space

(B) event

(C) parametric space

(D) random variable

10. A mixture of candies contains 6 mints, 4 toffees and 3 chocolates. If a person makes a random selection of one of these candies, find the probability of getting a toffee or chocolate :

(A) $\frac{6}{13}$

(B) $\frac{1}{13}$

(C) $\frac{6}{7}$

(D) $\frac{7}{13}$

11. If $P(A) = 0.3$, $P(B) = 0.78$ and $P(A \cup B) = 0.92$ then $P(\overline{A} \cap B)$ is :

(A) 0.62

(B) 0.55

(C) 0.64

(D) 0.48

12. An appliance store purchases electric ranges from two companies. From company A, 500 ranges are purchased and 2% are defective. From company B, 850 ranges are purchased and 2% are defective. Given that the range is defective, find the probability that it came from company B.

(A) 0.37

(B) 0.63

(C) 0.83

(D) 0.73

13. Let X be a random variable having pmf $P(X = x) = k(x + 1)$; $x = 0, 1, 2$ and 3. The value of k is :

(A) 10

(B) $\frac{1}{4}$

(C) $\frac{1}{10}$

(D) $\frac{1}{6}$

14. If X and Y are two independent binomial variables, then the conditional distribution of X given X+Y is :

(A) poisson

(B) binomial

(C) negative binomial

(D) hyper geometric

15. For negative binomial distribution :

(A) mean > variance

(B) mean < variance

(C) mean = variance

(D) mean = standard deviation

16. The variance of the distribution $\frac{e^{-5}5^x}{x!}$; x = 0, 1, 2, 3 ..... is :

(A) 5

(B) –5

(C) 25

(D) –25

17. Let X have uniform distribution over the interval (a,b), then variance of X is :

(A) $\frac{(b-a)^2}{2}$

(B) $\frac{(b-a)^2}{12}$

(C) $\frac{(b+a)}{2}$

(D) $\sqrt{ab}$

18. The value of $\Gamma 4$ is :

(A) 3

(B) 4

(C) 5

(D) 6

19. The mean of a beta distribution of first kind with parameters a and b is $\frac{1}{3}$, then :

(A) a = b

(B) 2a = b

(C) a = 2b

(D) a = 3b

20. For a standard normal variate, the value of mean is :

(A) 0

(B) 1

(C) ∞

(D) not defined

21. A random sample of 500 electric bulbs was taken from a large consignment and 65 were found to be defective. The standard error of the proportion of defective bulbs in a sample of this size is :

(A) 0.780

(B) 0.130

(C) 0.015

(D) 0.011

22. The probability of rejecting null hypothesis when alternative hypothesis is true is known as :

(A) power of test

(B) critical value

(C) type I error

(D) type II error

23. A hypothesis that completely specifies the population distribution is called :

(A) null hypothesis

(B) alternative hypothesis

(C) simple hypothesis

(D) composite hypothesis

24. If the critical region is located equally on both sides of the sampling distribution of test-statistic, the test is called :

(A) one tailed

(B) left tailed

(C) right tailed

(D) two tailed

25. The test used for testing the equality of standard deviations of two normal populations is :

(A) Z-test

(B) t-test

(C) F-test

(D) χ^2-test

26. In the test concerning mean of a normal population, t-test is used when :

(A) σ is known and n is large

(B) σ is unknown and n is large

(C) σ is known and n is small

(D) σ is unknown and n is small

27. Given the following information about two samples from two normal populations

$n_1 = 8, \sum (X - \overline{X})^2 = 94.5, \; n_2 = 10$ and

$\sum (Y - \overline{Y})^2 = 101.7$

The value of F statistic is :

(A) 1.195

(B) 0.837

(C) 1.162

(D) 0.935

28. The mean and variance of Chi-square distribution with n degrees of freedom are :

(A) n and 2n

(B) n and n - 1

(C) n and 2(n - 1)

(D) n - 1 and 2n - 1

29. If all the observations in a random sample are identical, then standard error will be :

(A) constant

(B) one

(C) between zero and one

(D) zero

30. In simple random sampling without replacement, $V(\overline{y})$ is given by :

(A) $\left(\frac{N-n}{nN}\right)S^2$

(B) $\left(\frac{N-1}{Nn}\right)S^2$

(C) $N^2\left(\frac{N-n}{n}\right)S^2$

(D) $\left(\frac{N-n}{N}\right)S^2$

31. In simple random sampling without replacement from a population of 400 units, the finite population correction is 0.75. The sample size is :

(A) 100

(B) 75

(C) 50

(D) 60

32. Compared to complete enumeration, sampling may be unable to provide :

(A) reduction in cost

(B) increase in administrative efficiency

(C) estimate of error

(D) measurement for the whole population

33. In usual notations, the relation among the variances in stratified random sampling is :

(A) $V_{ran} \leq V_{prop} \leq V_{opt}$

(B) $V_{opt} \leq V_{prop} \leq V_{ran}$

(C) $V_{prop} \leq V_{opt} \leq V_{ran}$

(D) $V_{opt} \leq V_{ran} \leq V_{prop}$

34. A sample of 50 students is to be drawn from a population consisting of 500 students belonging to two institutions A and B. The number of students in the institutions A and B are 200 and 300 respectively. The number of samples selected from institutions A and B by using proportional allocation method are :

(A) 25, 25

(B) 15, 35

(C) 20, 30

(D) 30, 20

35. If S^2_{wsy} denote the mean square among units which lie within the same systematic sample, the mean of a systematic sample is more precise than the mean of a simple random sampling, if and only if :

(A) $S^2_{wsy} < S^2$

(B) $S^2_{wsy} > S^2$

(C) $S^2_{wsy} = S^2$

(D) nothing definite

36. Determining the sample interval (represented by k), randomly selecting a number between 1 and k, and including each k^{th} element in your sample are the steps for which form of sampling ?

(A) Simple random sampling

(B) Stratified sampling

(C) Systematic sampling

(D) Cluster sampling

37. The basic principles of design of experiments are :

(A) replication

(B) randomization

(C) local control

(D) all of these

38. To remove one source of heterogeneity in the experimental units, we may use :

(A) CRD

(B) RBD

(C) LSD

(D) All of these

39. The analysis of variance technique was given by :

(A) R.A. Fisher

(B) Karl Pearson

(C) Irving Fisher

(D) P.C. Mahalanobis

40. In CRD, having n number of observations and v number of treatments, the error degree of freedom is :

(A) $n-1$

(B) $v-1$

(C) $n-v$

(D) $nv-1$

41. In the optimal simplex table, $C_j - Z_j = 0$ indicates :

(A) unbounded solution

(B) alternate solution

(C) cycling

(D) infeasible solution

42. Graphical method can be applied to solve a LPP if there are only ..... variables.

(A) 5

(B) 4

(C) 3

(D) 2

43. When the total supply is equal to total demand in a transportation problem, the problem is said to be :

(A) balanced

(B) unbalanced

(C) degenerate

(D) none of these

44. A solution that can indefinitely increase or decrease the values of objective function of the linear programming problem is known as :

(A) feasible solution

(B) optimum solution

(C) unbounded solution

(D) unique solution

45. Increase in the number of patients in the hospital due to heat stroke is :

(A) secular trend

(B) cyclic variation

(C) seasonal variation

(D) irregular variation

46. An orderly set of data arranged in accordance with their time of occurrence is called :

(A) arithmetic series

(B) geometric series

(C) harmonic series

(D) time series

47. A period for which index number is determined is called :

(A) base period

(B) current period

(C) normal period

(D) time period

48. If the Laspeyre's price index is 324 and Paasche's price index 144, then Fisher's ideal index is :

(A) 216

(B) 234

(C) 180

(D) 468

49. Which of the following is not a measure of fertility ?

(A) crude birth rate

(B) total fertility rate

(C) age specific fertility rate

(D) age specific death rate

50. Which measure is used to calculate the rate of natural increase in a population ?

(A) birth rate minus death rate

(B) fertility rate minus mortality rate

(C) death rate minus birth rate

(D) marriage rate minus divorce rate

51. The number of births per thousand women of child bearing age is :

(A) crude birth rate

(B) general fertility rate

(C) total fertility rate

(D) gross reproduction rate

52. Vital statistics is mainly concerned with :

(A) births

(B) deaths

(C) marriages

(D) all of these

53. Faults due to assignable causes :

(A) can always be removed

(B) can sometimes be removed

(C) can't be removed

(D) none of these

54. Process control is carried out :

(A) before production

(B) after production

(C) during production

(D) none of these

55. Which probability distribution is used to construct the c-chart ?

(A) Normal

(B) Exponential

(C) Poisson

(D) Binomial

56. Identify the device through which data and instructions are entered into a computer ?

(A) Software

(B) Memory

(C) Output device

(D) Input device

57. Let $(X_1, X_2, \ldots\ldots X_n)$ be a random sample from a $N(\mu, 1)$ population, then $T = \frac{1}{n}\sum_{i=1}^{x} X_i^2$ is an unbiased estimator of :

(A) μ^2

(B) $\mu^2 + 1$

(C) $\frac{\mu^2}{n}$

(D) $\frac{\mu^2}{2}$

58. An estimator which provides all the information provided by a sample with respect to the parameter is called :

(A) unbiased

(B) consistent

(C) efficient

(D) sufficient

59. Let $(X_1, X_2, \ldots\ldots X_n)$ be a random sample from Poisson (λ). Then the moment estimator of λ is :

(A) $\overline{X}$

(B) $\sum_{i=1}^{n} X_i$

(C) $\frac{1}{\overline{X}}$

(D) all of these

60. Which of the following represents confidence coefficient ?

(A) α

(B) $\alpha/2$

(C) $1 - \alpha$

(D) $1 - \beta$

Sr. No. .....

ENTRANCE TEST-2023

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

OPTION—1 : STATISTICS

OPTION—2 : MATHEMATICS FOR STATISTICS

Question Booklet Series **A**

Total Questions : 60

Time Allowed : 70 Minutes

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be **'Negative Marking'** for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer Sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/ herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

OPTION—1 : STATISTICS

1. The coefficient of variation is 58%. If mean is 10 then standard deviation is :
   (A) 5.8
   (B) 580
   (C) 0.17
   (D) None of these
2. If the distribution is negatively skewed, then :
   (A) Mean is more than the mode
   (B) Mean is less than the mode
   (C) Median is at right to the mode
   (D) Mean is at right to the median
3. The Median of scores 25, 45, 35, 35, 40, 30 is :
   (A) 45
   (B) 40
   (C) 35
   (D) 30
4. The arithmetic mean of the first ten whole numbers is :
   (A) 5.5
   (B) 5
   (C) 4
   (D) 4.5
5. The correlation coefficient between two variables X and Y is 0.4. The correlation between 2X and (–Y) will be :
   (A) 0.4
   (B) –0.8
   (C) –0.4
   (D) 0.8
6. In regression analysis, the variable that is used to explain the change in the outcome of an experiment is called :
   (A) The independent variable
   (B) The predictor variable
   (C) The explanatory variable
   (D) All of the above
7. If the coefficient of determination is equal to 1. Then correlation coefficient :
   (A) Must also be equal to 1
   (B) Can either be –1 or +1
   (C) Can be any value between –1 to 1
   (D) Must be –1
8. If r = 0.8, $b_{yx} = 1.5$, then b_{xy} is approximately :
   (A) 0.32
   (B) 0.42
   (C) 0.75
   (D) 1
9. Suppose P(X) = 0.36 and P(Y) = 0.41. If P(X|Y) = 0.27, what is P(Y|X) ?
   (A) $\frac{(0.36)(0.41)}{(0.27)}$
   (B) $\frac{(0.27)(0.41)}{(0.36)}$
   (C) $\frac{(0.36)(0.27)}{(0.41)}$
   (D) $\frac{(0.27)}{0.36 + 0.41}$
10. If P(A) = 0.32 and P(B) = 0.45, what is $P(A \cup B)$ if A and B are independent ?
    (A) 0.144
    (B) 0.626
    (C) 0.770
    (D) 0.856
11. The probability of throwing 10 with 2 dice is :
    (A) 1/6
    (B) 1/12
    (C) 2/3
    (D) 1/4

12. Which of the following can be the probability of an event ?
(A) –1.3
(B) 004
(C) 3/8
(D) 10/7

13. Let X have pmf

$$f(x) = \frac{x}{10},\ x = 1,2,3,4$$

Then E(X) is equal to :
(A) 3
(B) 6
(C) 9
(D) 12

14. Given $E(X + 4) = 10$ and $E[(X + 4)^2] = 116$. Then $V(X + 4)$ is :
(A) 4
(B) 16
(C) 3
(D) 9

15. The variance of probability distribution

X	–3	–2	–1	0	1	2	3
P(X)	1/7	1/7	1/7	1/7	1/7	1/7	1/7

is :
(A) 0
(B) 8
(C) 4
(D) 12

16. The Mean of the distribution

$$\binom{10}{x}\left(\frac{2}{5}\right)^x\left(\frac{3}{5}\right)^{10-x},\ x = 0,1,2,\ldots 10$$

is :
(A) 4
(B) 5
(C) 6
(D) 10

17. Let X have the Uniform pdf U(0, 100), then variance of X is given by :
(A) 1/12
(B) 100/12
(C) 1000/12
(D) 10000/12

18. The point of inflexion of normal curve are :
(A) $\mu \pm \sigma$
(B) $\mu \pm 3\sigma$
(C) $\mu \pm 2\sigma$
(D) None of these

19. Match the following :

Distribution		**MGF**
a.	Normal	1. $e^{\lambda}(e^t - 1)$
b.	Gamma	2. $e^{\mu t} + t^2\sigma^2/2$
c.	Poisson	3. $(1 - t)^{-\lambda}$

(A) a–2, b–1, c–4
(B) a–3, b–1, c–2
(C) a–1, b–3, c–2
(D) a–2, b–3, c–1

20. If $X_1, X_2, X_3, \ldots\ldots X_n$ are independent and have normal distributions $N(\mu_i, \sigma_i^2)$, i = 1, 2, 3 ..... n, respectively. Then the distribution of

$$W = \sum_{i=1}^{n}\left(\frac{x_i - \mu_i}{\sigma_i}\right)^2 \text{ is :}$$

(A) Gamma distribution
(B) Chi-square distribution
(C) Normal distribution
(D) t-distribution

21. In a test with a standard deviation of 12 and mean 44 a student scored 41 marks. His Z score is :
(A) 0.50
(B) –0.50
(C) 0.25
(D) –0.25

22. Arrange the following steps in process of hypothesis testing in proper sequence :

a. Select the level of significance

b. Setup null and alternative hypothesis

c. Establish the decision rule

d. Perform computation

e. Select test statistics

f. Draw decision

(A) a, b, c, d, e, f

(B) a, b, e, d, c, f

(C) b, a, c, d, e, f

(D) b, a, e, c, d, f

23. The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected is said to be :

(A) Critical region

(B) Significance value

(C) Critical value

(D) Acceptance region

24. Which of the following statements best describes on type I error ?

(A) Rejecting a null hypothesis when it is true

(B) Failing to reject a false null hypothesis

(C) Accepting a true alternative hypothesis

(D) Rejecting a false alternative hypothesis

25. For testing of goodness of fit :

(A) The expected frequency should exceed 5

(B) The observed frequency should exceed 5

(C) Both the expected and observed frequency should exceed 5

(D) None of the above conditions are necessary

26. What type of data do you need for a Chi square test ?

(A) Ordinal

(B) Interval

(C) Ratio

(D) Categorical

27. A Medical Assistant sampled the blood pressure of 20 randomly selected patients with high blood pressure and after receive a dose of a new medicine, which hypothesis test should she run ?

(A) t-test for single mean

(B) F-test for equality of variances

(C) Independent t-test for difference of means

(D) Paired t-test

28. Which of the following are conditions for using the t-distribution for small sample difference tests ?

I. Samples must be independent.

II. Samples must be drawn from normal populations.

III. Samples must be of equal size.

(A) I only

(B) I and II

(C) II only

(D) I and III

29. The complete list of population, where each sampling unit is identified by a number is known as :

(A) Voter list

(B) Sampling frame

(C) A list of random numbers

(D) None of these

30. The relative efficiency of SRSWOR with SRSWR is :

(A) $\frac{N - n}{N - 1}$

(B) $\frac{N - n}{N}$

(C) $\frac{n}{N}$

(D) $\frac{N - 1}{N}$

31. In simple random sampling, the sample mean is :
    (A) Always zero
    (B) Smaller than population mean
    (C) Equal to population mean
    (D) Random variable

32. The finite population correction in usual notation is expressed as :
    (A) $(N - n)/N$
    (B) $1 - (n/N)$
    (C) Both (A) and (B)
    (D) None

33. Which of the following is not true ?
    (A) $Var(\bar{y}_{st})_P \geq Var(\bar{y}_{st})_{Ney}$
    (B) $Var(\bar{y})_{SRS} \geq Var(\bar{y}_{st})_P$
    (C) $Var(\bar{y})_{SRS} \leq Var(\bar{y}_{st})_{Ney}$ b)
    (D) All of these

34. In proportional allocation we have :
    (A) $n_i = \frac{n}{N} N_i$
    (B) $n_i = n/k$
    (C) $n_i = N/k$
    (D) $n_i = \frac{n}{N_i} N$

35. Which of the following is an example of systematic sampling ?
    (A) A researcher selects every 10^{th} person who enters a shopping mall to participate in a study
    (B) A researcher selects a random sample of participants from a list of registered voters
    (C) A researcher selects a convenience sample of participants from a local community center
    (D) None of these

36. The purpose of stratified sampling is to :
    (A) To ensure that the sample is representative of the population
    (B) To save time and money by not having to sample the entire population
    (C) To obtain a biased sample
    (D) To obtain non-random sample

37. For analysing the completely randomised design with t treatments each replicated r times each, with one missing observation, total degree of freedom are :
    (A) Rt
    (B) rt – 1
    (C) rt – 2
    (D) (r – 1) (t – 1)

38. If the total degrees of freedom and between treatments degrees of freedom in a completely randomized design are 15 and 4 respectively, the degrees of freedom for error will be :
    (A) 11
    (B) 14
    (C) 18
    (D) 19

39. How many factors are involved in a randomized block design ?
    (A) One
    (B) Two
    (C) Three
    (D) Four

40. In ANOVA, what is the null hypothesis ?
    (A) There is no difference between the means of the groups
    (B) There is a difference between the means of the groups
    (C) The groups are not normally distributed
    (D) The sample size is too small to draw a conclusion

41. At every iteration of simplex method, for a minimization problem, a variable in current basis is replaced with another variable that has :
(A) A negative value of $Z_j - C_j$
(B) The value of $Z_j - C_j = 0$
(C) A positive value of $Z_j - C_j$
(D) None

42. A set of values of decision variables which satisfies the linear constraints and non-negativity conditions of a LPP is known as :
(A) Solution
(B) Feasible solution
(C) Optimal solution
(D) Unbounded solution

43. The optimal value of the objective function is attained at the points :
(A) Given by intersection of lines representing inequations with axes only
(B) Given by intersection of lines representing inequations with X-axis only
(C) Given by corner points of the feasible region
(D) At the origin

44. If the two constraints do not intersect in the positive quadrant of the graph, then :
(A) The problem is infeasible
(B) The solution is unbounded
(C) One of the constraints is redundant
(D) None of these

45. In fitting a straight line, the value of slope b remains unchanged with the change of :
(A) Scale
(B) Origin
(C) Both (A) and (B)
(D) Neither (A) Nor (B)

46. Secular trend is indicative of long-term variation towards :
(A) Increase only
(B) Either increase or decrease
(C) Decrease only
(D) None of these

47. The trend values in freehand curve method are obtained by :
(A) Equation of straight line
(B) Second degree parabola
(C) Graph
(D) All of these

48. For the given data semi averages for the second half is given by :

Year	2010	2011	2012	2013	2014	2015	2016	2017
Sales	20	16	9	11	40	23	21	12

(A) 14
(B) 22
(C) 23
(D) 24

49. The rate computed by adding the age specific fertility rates of various age groups of child bearing age is known as :
(A) Crude Birth Rate
(B) Net Reproduction Rate
(C) Total Fertility Rate
(D) General Fertility Rate

50. In a given year, the Crude Birth Rate of population of size 1,50,000 is 12. The number of births is :
(A) 18
(B) 180
(C) 1800
(D) 18000

51. The population will increase, remains stationary or decreasing according to whether the NRR exceeds, equal or is less than :
(A) 0
(B) 0.5
(C) 1
(D) 1.5

52. The data related to births, deaths, marriage and divorce is called :
(A) Mortality
(B) Morbidity
(C) Vital statistics
(D) Survey

53. If the sample values lie within the control limits and are in a random way, we say that the process is :

(A) Under control

(B) Under warning limits

(C) Out of control

(D) Need to stop the process

54. If a characteristic follows normal distribution, then 3 sigma limits covers _____ of observations.

(A) 0.27%

(B) 68.26%

(C) 95.44%

(D) 99.73%

55. The chart used to monitor attributes is :

(A) Range chart

(B) C-chart

(C) Mean chart

(D) All of the above

56. Which of the following is a type of control chart used in Statistical Quality Control ?

(A) Histogram

(B) Box and Whisker Plot

(C) Scatter Plot

(D) $\overline{X}$ chart

57. What is estimation in Statistics ?

(A) The process of making a prediction or approximation about a population parameter based on a sample of data

(B) The process of accurately measuring a population parameter

(C) The process of collecting data on a population

(D) The process of analysing data to draw conclusions

58. A confidence interval will be widened if :

(A) The confidence level is decreased and the sample size is increased

(B) The confidence level is increased and the sample size is reduced

(C) The confidence level is increased and the sample size is increased

(D) The confidence level is decreased and the sample size is decreased

59. Sampling distribution is :

(A) A distribution of the population parameters

(B) A distribution of the sample statistics

(C) A distribution of the individual data points in sample

(D) A distribution of the differences between the sample and population parameters

60. A function for estimating a parameter is called as :

(A) Estimate

(B) Estimation

(C) Estimator

(D) None of these

OR

OPTION—2 : MATHEMATICS FOR STATISTICS

1. If

$$f(x) = \frac{x}{|x|},\ x \neq 0$$

$$= 0\ x = 0,$$

then $\lim_{x \to 0} f(x) =$

(A) 0

(B) 1

(C) –1

(D) None of these

2. What type of discontinuity does the function $f(x) = e^{-\frac{1}{x}}$ have at x = 0 ?

(A) Removable discontinuity

(B) Discontinuity of the first kind

(C) Discontinuity of the second kind

(D) None of these

3. Which of the following is true for the function $y = \log\left(x + \sqrt{1 + x^2}\right)$?

(A) $xy_2 + y_1 = 0$

(B) $(1 + x^2)y_2 + y_1 = 0$

(C) $(1 + x^2)y_2 + xy_1 = 1$

(D) $(1 + x^2)y_2 + xy_1 = 0$

4. If $u = \cos^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) cos u

(B) 2 cot u

(C) $\frac{1}{2}\cot u$

(D) $-\frac{1}{2}\cot u$

5. The angle of intersection of the curves $r = a\theta$ and $r = \frac{a}{\theta}$ is :

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{2}$

6. The angle between the radius vector and the tangent to the curve $r = a(1 - \cos\theta)$ at any point is equal to :

(A) θ

(B) 2θ

(C) $\frac{\theta}{2}$

(D) $\frac{\theta}{3}$

7. The radius of curvature of the curve $x^2 + y^2 = 2$ at the point (1, 1) is :

(A) $\sqrt{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) 2

(D) $\frac{1}{2}$

8. The number of asymptotes to the curve

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

is :

(A) 2

(B) 1

(C) 4

(D) None

9. Which of the following is/are true for the function $f(x) = \sin x,\ x \in I = \left[0, \frac{\pi}{2}\right]$?

(A) There is a value of $x \in I$ where $f(x) = \frac{1}{3}$

(B) The maximum value of f(x) in I is 1

(C) There is a value of $x \in I$ where $f'(x) = 0$

(D) All of the above

10. The coefficient of x^n in the Maclaurin's series of $-\log(1 - x)$ is :

(A) $\frac{1}{n}$

(B) $-\frac{1}{n}$

(C) $\frac{1}{n!}$

(D) $-\frac{1}{n!}$

11. If $f(x) = x^2 - 3x - 1,\ x \in [1, 3]$, then the value of the "c" of the Mean Value Theorem is :

(A) 2.75

(B) 2.5

(C) 2.3

(D) 2.1

12. The value of $\lim_{x \to +\infty}(1 + x)^{\frac{1}{x}}$ is :

(A) 0

(B) 1

(C) e

(D) $\frac{1}{e}$

13. The value of $\int_0^1 \frac{1}{2e^x - 1}\,dx$ is :

(A) log 2

(B) $\log\left(\frac{2e - 1}{e}\right)$

(C) 1

(D) 0

14. The integral of $\sqrt{x}\ \tan^{-1}\sqrt{x}$ with respect to x is equal to :

(A) $x^{\frac{3}{2}} \tan^{-1}\sqrt{x} - \frac{x}{2} + \frac{1}{2}\log(1 + x) + c$

(B) $\frac{3}{2}\left[x^{\frac{3}{2}} \tan^{-1}\sqrt{x} - \frac{x}{2} + \frac{1}{2}\log(1 + x)\right] + c$

(C) $\frac{2}{3}\left[x^{\frac{3}{2}} \tan^{-1}\sqrt{x} - \frac{x}{2} + \frac{1}{2}\log(1 + x)\right] + c$

(D) $2\left[x^{\frac{3}{2}} \tan^{-1}\sqrt{x} - \frac{x}{2} + \frac{1}{2}\log(1 + x)\right] + c$

15. Which of the following is/are true for $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$?

(A) $n(I_{n-1} + I_{n-2}) = 1$

(B) $I_n + I_{n-2} = \frac{1}{n-1}$

(C) $I_1 = \log\sqrt{2}$

(D) All of the above

16. What is the value of $\int_0^{\infty} \frac{dx}{(1+x^2)^2}$?

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{2}$

(D) $\frac{3\pi}{4}$

17. Which of the following substitutions reduces the differential equation $xy(1+xy^2)\frac{dy}{dx} = 1$ to a linear differential equation ?

(A) $\frac{1}{x} = z$

(B) $\frac{1}{y} = z$

(C) $xy = z$

(D) $y = x^2$

18. Which of the following is a necessary and sufficient condition for the differential equation $M\,dx + N\,dy = 0$ to be exact ?

(A) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(C) Neither (A) Nor (B)

(D) Both (A) and (B)

19. Which of the following is the particular integral of the differential equation $(D^3 - 3D^2 + 4)y = e^{2x}$?

(A) x^2e^{2x}

(B) $\frac{x}{6}e^{2x}$

(C) $\frac{x^2}{6}e^{2x}$

(D) $\frac{x^2}{2}e^{2x}$

20. The complementary function of the differential equation

$$(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$

is :

(A) $c_1(3x+2)^2 + \frac{c_2}{(3x+2)^2}$

(B) $c_1(3x+2) + \frac{c_2}{(3x+2)}$

(C) $c_1(3x+2)^2 + c_2(3x+2)$

(D) $c_1(3x+2) + \frac{c_2}{(3x+2)^2}$

21. The general solution of the differential equation $p^2 + 2py \cot x = y^2$ is given by :

(A) $\frac{y}{1 \pm \cos x} = c$

(B) $y(1 \pm \cos x) = c$

(C) $x(1 \pm \cos y) = c$

(D) $y(1 \pm \sin x) = c$

22. The general solution of the differential equation $p = \tan\left(x - \frac{p}{1+p^2}\right)$ is given by :

(A) $x = \tan^{-1} p + \frac{p}{1+p^2}, y = c - \frac{1}{1+p^2}$

(B) $x = c - \frac{1}{1+p^2}, y = \tan^{-1} p + \frac{p}{1+p^2}$

(C) Neither (A) Nor (B)

(D) Both (A) and (B)

23. The solution of the differential equation

$yp + px - p^2x - y = p$

is given by :

(A) $y = x + c$

(B) $y = \frac{x^2}{2} + c$

(C) $y = cx + \frac{c}{c-1}$

(D) $y = c\,x$

24. The substitution $X = x^2$, $Y = y^2$ reduces the equation $(px - y)(py + x) = 2p$ to :

(A) Linear form

(B) Bernoulli's form

(C) Clairut's form

(D) None of these

25. The values of the Bessel's functions $J_0(x)$ and $J_1(x)$ at $x = 0$ are respectively :

(A) 0 and 1

(B) 1 and 0

(C) 0 and 0

(D) 1 and 1

26. Which of the following is true ?

(A) $J_2(x) = J_1(x) - J_0(x)$

(B) $J_2(x) = J_1(x) + J_0(x)$

(C) $J_2(x) = \frac{x}{2} J_1(x) - J_0(x)$

(D) $J_2(x) = \frac{2}{x} J_1(x) - J_0(x)$

27. Which of the following is not true for the Legendre polynomial $P_n(x)$?

(A) $P_n(1) = 1$

(B) $P_n(-x) = (-1)^n P_n(x)$

(C) $P_{2n+1}(0) = 0$

(D) $P_{2n}(0) = \frac{(-1)^n (2n)!}{(n!)^2}$

28. For $m = n$, $\int_0^1 P_m(x) P_n(x)\, dx =$

(A) 0

(B) 1

(C) $\frac{2}{2n+1}$

(D) $\frac{2}{n+1}$

29. The order and the degree of the partial differential equation $\frac{\partial^2 u}{\partial x \partial y} = \left(\frac{\partial u}{\partial z}\right)^3$ are respectively :

(A) 2 and 2

(B) 2 and 1

(C) 1 and 2

(D) 2 and 3

30. The solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when x = 0, $z = e^y$ and $\frac{\partial z}{\partial y} = 1$, is :

(A) $z = e^x \cos y + \sin y$

(B) $z = e^y \cos y + \sin x$

(C) $z = e^y \cos x + \sin x$

(D) $z = e^x \sin y + \cos x$

31. The general solution of the partial differential equation $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ is :

(A) $x^2 + y^2 + z^2 = f(x + y + z)$

(B) $xyz = f(x^2 + y^2 + z^2)$

(C) $f\left(x^2 + y^2 + z^2, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$

(D) $f\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, xyz\right) = 0$

32. The complete solution of partial differential equation $p = e^q$ is :

(A) $z = ax + y \log a + c$

(B) $z = ax + \log y + c$

(C) $z = ax + by + c$

(D) $z = ay + x \log a + c$

33. Which of the following sets is not countable ?

(A) The set of natural numbers

(B) The set of rational numbers

(C) The set of complex numbers with rational real and imaginary parts

(D) The set of points in the closed interval [1.2]

34. Which of the following is a bounded set ?

(A) The set of natural numbers

(B) The set of rational numbers

(C) The set $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\right\}$

(D) The set of positive real numbers

35. Which of the following is true ?

(A) Every infinite set has a limit point

(B) Every bounded set has a limit point

(C) Every finite set has a limit point

(D) Every infinite bounded set has a limit point

36. If for any real numbers x, y and a, $|x - a| < \varepsilon$, $|y - a| < \varepsilon$, then $|x - y| <$

(A) ε

(B) $\frac{\varepsilon}{2}$

(C) 2ε

(D) $a + \varepsilon$

37. For what value of a_n the sequence $\{a_n\}$ oscillates infinitely ?

(A) $a_n = (-1)^n$

(B) $a_n = (-1)^n n$

(C) $a_n = n^2$

(D) $a_n = -2^n$

38. The limit of the sequence $\{a_n\}$, where

$$a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + ... + \frac{1}{3^n},$$

is :

(A) 1

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

39. For what of a_n is the sequence $\{a_n\}$ a monotonic decreasing sequence that is not convergent ?

(A) $a_n = \frac{1}{n}$

(B) $a_n = -n$

(C) $a_n = \frac{n+1}{n}$

(D) $a_n = \frac{n}{n^2+1}$

40. If b and c are positive real numbers, then the sequence $\left\{\left(1 + \frac{b}{n}\right)^{cn}\right\}$ converges to :

(A) e

(B) e^b

(C) e^c

(D) e^{bc}

41. Which of the following is not true ?

(A) A positive term series either converges or diverges to $+\infty$

(B) The series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if $\lim_{n \to \infty} = 0$

(C) The series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the sequence of its partial sums is convergent

(D) The sum of two convergent series is convergent

42. For what value of x is the series

$1 + x + x^2 + x^3 + ...$

divergent ?

(A) $x < -1$

(B) $x = -1$

(C) $x \geq 1$

(D) $-1 < x < 1$

43. For what value of x is the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$ convergent ?

(A) $x < 1$

(B) $x > 1$

(C) $x = 1$

(D) None of these

44. Which of the following is true for the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n = \frac{(-1)^{n-1}}{n}$ and $b_n = \frac{(-1)^{n-1} n}{2n-1}$?

(A) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent

(B) Neither $\sum_{n=1}^{\infty} a_n$ Nor $\sum_{n=1}^{\infty} b_n$ is convergent

(C) $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is divergent

(D) $\sum_{n=1}^{\infty} a_n$ is divergent but $\sum_{n=1}^{\infty} b_n$ is convergent

45. Which of the following series is/are uniformly convergent for all real values of x and $0 < r < 1$?

(A) $\sum_{n=1}^{\infty} r^n \cos nx$

(B) $\sum_{n=1}^{\infty} r^n \sin nx$

(C) $\sum_{n=1}^{\infty} r^n \cos n^2 x$

(D) All of the above

46. Which of the following is true for the sequence $\{f_n\}$, where $f_n(x) = \tan^{-1}(nx)$, $x \geq 0$, $n = 1, 2, 3, \ldots$?

(A) $\{f_n\}$ is not uniformly convergent in any interval [a, b], $a > 0$

(B) $\{f_n\}$ is uniformly convergent in [0, b]

(C) $\{f_n\}$ is not point-wise convergent in [0, b]

(D) None of these

47. Which of the following is not true for the sequence $\{f_n\}$, where $f_n(x) = nxe^{-nx^2}$, $n = 1, 2, 3, \ldots$ and $f(x) = \lim_{n \to \infty} f_n(x)$?

(A) $\int_0^1 f(x)\, dx = 0$

(B) $\lim_{n \to \infty} \int_0^1 f_n(x)\, dx = \frac{1}{2}$

(C) $f(x) = 0, \forall x$

(D) $\{f_n\}$ does not converge uniformly to f

48. The radius of convergence of the power series $1 + 2x + 3x^2 + 4x^3 + \ldots\ldots$ is :

(A) 1

(B) ∞

(C) 0

(D) 2

49. Which of the following is a group ?

(A) The set of natural numbers under addition

(B) The set of real numbers under addition

(C) The set of real numbers under multiplication

(D) The set of complex numbers under multiplication

50. Which of the following conditions makes any multiplicative group G into an Abelian group ?

(A) $\forall a, b \in G, (a \cdot b)^2 = a^2 \cdot b^2$

(B) Each element of G is idempotent

(C) Each element of G is its own inverse

(D) All of the above

51. What is the order of the symmetric group S_4 ?

(A) 4

(B) 6

(C) 24

(D) 120

52. The number of elements in the quaternion group is equal to :

(A) 4

(B) 6

(C) 8

(D) 24

53. Which of the following is not true ?

(A) A non-empty subset S of a group G is a subgroup of G if and only if $\forall a, b \in S, ab^{-1} \in S$

(B) For any subgroup S of a group G, O(S)|O(G)

(C) For any element a in a group G, $a^{O(G)} = e$

(D) For any two subgroups A and B of G, $A \cup B$ is also a subgroup of G

54. Which of the following is a generator of the cyclic group formed by the n^{th} roots of unity ?

(A) $e^{\frac{2\pi i}{n}}$

(B) $e^{\frac{\pi i}{n}}$

(C) $e^{2n\pi}$

(D) None of these

55. Which of the following is not true for the multiplicative group G = {1, –1, i, –i}, where $i = \sqrt{-1}$?

(A) G is an abelian group

(B) G is a cyclic group with two generators

(C) O(i) = 4

(D) G has no subgroup of order 2

56. The order of the quotient group $\frac{Z}{N}$, where Z is the additive group of integers and $N = \langle 3 \rangle$, the subgroup of Z consisting of all multiples of 3, is :

(A) 1

(B) 2

(C) 3

(D) Infinity

57. Which of the following rings is not free from zero divisors ?

(A) Ring of integers

(B) Ring of all square matrices of order 2

(C) Ring of integers modulo a prime number

(D) Ring of complex numbers

58. If f and g are any two non-zero polynomials over a ring R without proper zero divisors, then degree(f.g) :

(A) is less than max(deg f, deg g)

(B) is less than deg f + deg g

(C) is equal to deg f + deg g

(D) is equal to deg f. deg g

59. Which of the following is not true ?

(A) Every finite integral domain is a field

(B) Every field is an integral domain

(C) Every ideal of a ring R is a subring of R

(D) The ring of integers is an ideal of the ring of rational numbers

60. Which of the following is true for any element a in a ring R and the subset $S = \{x \in R; xa = 0\}$?

(A) S is a subring of R

(B) S is a right ideal of R

(C) S is an ideal of R

(D) None of these

ROUGH WORK

Sr. No. 133

ENTRANCE TEST-2022

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

OPTION—1 : STATISTICS

OPTION—2 : MATHEMATICS FOR STATISTICS

Total Questions : 60
Time Allowed : 70 Minutes

Question Booklet Series A

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be **'Negative Marking'** for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer Sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/ herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

SEAL

OPTION—1 : STATISTICS

1. The Bowley's coefficient of skewness ranges between :

   (A) 0 and 1

   (B) –1 and 0

   (C) –3 and 3

   (D) –1 and 1

2. A manager of a departmental store wants to place orders of the most economical size of every item. Which measure of central tendency can help him in arriving at the desired size ?

   (A) Arithmetic mean

   (B) Geometric mean

   (C) Mode

   (D) Median

3. If a constant is added to every item of a series, then which one of the following measures of dispersion would change ?

   (A) Mean deviation about mean

   (B) Range

   (C) Standard deviation

   (D) Coefficient of variation

4. If the geometric mean and harmonic mean of two numbers are respectively 28 and 16, then the arithmetic mean of these two numbers is :

   (A) 49

   (B) 28

   (C) 22

   (D) 16

5. If the correlation coefficient between two variables X and Y is 0.8, then the correlation coefficient between U = X – 0.4 and V = Y – 0.3 is :

   (A) 0.1

   (B) 0.4

   (C) 0.5

   (D) 0.8

6. The law of demand states that the price and demand of commodity are inversely related. The correlation coefficient between these two economic variables would be :

   (A) Positive

   (B) Negative

   (C) Zero

   (D) Nothing definite

7. If $\sigma_x^2 = 4\sigma_y^2$ and r = 0.25, then the regression coefficient b_{xy} is :

   (A) 0.125

   (B) 0.0625

   (C) 0.50

   (D) 1.00

8. For obtaining the regression line of X on Y :

   (A) $E(Y - a - bX)^2$ is minimized

   (B) $E(Y - a - bX)^2$ is maximized

   (C) $E(X - a - bY)^2$ is minimized

   (D) $E(X - a - bY)^2$ is maximized

SEAL

9. A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If E is the event that a number less than 4 occurs on a single toss of the die, then P(E) :

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{1}{9}$

(D) $\frac{4}{9}$

10. The probability that a regularly scheduled flight departs on time is P(D) = 0.83; the probability that it arrives on time is P(A) = 0.82; and the probability that it departs and arrives on time is P (D∩A) = 0.78. Find the probability that a plane arrives on time, given that it departed on time :

(A) 0.95

(B) 0.94

(C) 0.50

(D) 0.24

11. In which theory of probability, the outcomes of a random experiment are assumed to be equally likely ?

(A) Statistical

(B) Subjective

(C) Classical

(D) Axiomatic

12. If A and B are any two events, then :

(A) $P(A \cap B) = P(A)\, P(A|B)$

(B) $P(A \cap B) = P(A)\, P(B|A)$

(C) $P(A \cup B) = P(A)\, P(B|A)$

(D) $P(A \cap B) = P(A|B)$

13. If the moment generating function of a random variable X is given by $M_x(t) = \frac{3}{3-t}$ then its standard deviation is given by :

(A) 1/3

(B) 2/9

(C) 1/5

(D) 1/9

14. Let X and Y be two independent Poisson variates. Then the conditional distribution of X given X + Y is :

(A) Poisson

(B) Geometric

(C) Binomial

(D) Negative binomial

15. The density function of a random variable X is given by $f(x) = \begin{cases} \frac{1}{2}x & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

The expected value of X is then :

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{4}{3}$

16. The mean of a binomial distribution is 20 and its standard deviation is 4. Then the number of trials is :

(A) 75

(B) 100

(C) 150

(D) 200

17. In an exponential distribution with mean other than unity :

(A) Mean = (variance)^2

(B) Mean = 2 variance

(C) Mean = standard deviation

(D) Mean = variance

18. Let Y have uniform distribution over the interval (α, β), then its mean is given by :

(A) $\frac{\beta+\alpha}{2}$

(B) $\frac{\beta-\alpha}{2}$

(C) $\frac{\beta+\alpha}{12}$

(D) $\frac{(\beta-\alpha)^2}{12}$

19. If $X \sim N(\mu, \sigma^2)$, then moment generating function of a standard normal variate is :

(A) $e^{t^2/2}$

(B) $e^{i\mu t+\sigma^2 t^2/2}$

(C) $e^{i\mu t+\sigma^2 t^2/2}$

(D) $e^{i\mu t-\sigma^2 t^2/2}$

20. For gamma distribution with single parameter :

(A) Mean > variance

(B) Mean < variance

(C) Mean = variance

(D) Mean = standard deviation

21. The probability distribution of a statistic i called :

(A) Statistic

(B) Standard error

(C) Frequency distribution

(D) Sampling distribution

22. Let $\underline{X} = (X_1, X_2, ..., X_n)$ be a sample and ω be th critical region. The power of test is defined by :

(A) $P(\underline{X} \in \omega | H_0)$

(B) $P(\underline{X} \in \omega | H_1)$

(C) $P(\underline{X} \in \varpi | H_0)$

(D) $P(\underline{X} \in \varpi | H_1)$

23. A company manufacturing electric bulbs clair that the average life of its bulbs is 1600 hou The average life and standard deviation of random sample of 100 such bulbs were 1570 hou and 120 hours respectively, then standard error the average life of bulbs is :

(A) 12 hours

(B) 1.2 hours

(C) 157 hours

(D) 15.7 hours

SEAL

24. The standard deviation of the sampling distribution of a statistic is known as :

(A) Standard error

(B) Mean squared error

(C) Variance

(D) Coefficient of variation

25. Consider the 2×2 contingency table on two attributes A and B :

	A_1	A_2
B_1	10	20
B_2	30	40

The value of χ^2 for testing the independence of attributes A and B is :

(A) 0.85

(B) 0.83

(C) 0.81

(D) 0.79

26. The range of F-variate is :

(A) $-\infty$ to ∞

(B) 0 to ∞

(C) 0 to 1

(D) –1 to 1

27. Goodness of fit of a distribution is tested by :

(A) Z test

(B) F test

(C) t-test

(D) Chi-square test

28. The degree of freedom for paired t-test based on n pairs of observations is :

(A) $n-2$

(B) $2(n-1)$

(C) $n-1$

(D) $2n-1$

29. Compared to complete enumeration, sampling may be unable to provide :

(A) Reduction in cost

(B) Increase in administrative efficiency

(C) Estimator of error

(D) Measurement of the whole population

30. In SRSWOR, the $V(N\bar{y})$ is given by :

(A) $\left(\frac{N-n}{nN}\right)S^2$

(B) $N\left(\frac{N-n}{n}\right)S^2$

(C) $N^2\left(\frac{N-n}{n}\right)S^2$

(D) $n\left(\frac{N-n}{N}\right)S^2$

31. The total number of possible samples of size n from a population of size N in case of SRSWOR is :

(A) $\binom{N}{n}$

(B) N^n

(C) $\left(\frac{N}{n}\right)^n$

(D) $\frac{1}{N^n}$

32. The difference between a sample estimate and the population parameter from a complete count is termed as :

(A) Human error

(B) Sampling error

(C) Non-sampling error

(D) Mistakes

33. Stratified sampling is appropriate when the population is :

(A) Heterogeneous

(B) Homogeneous

(C) Finite

(D) Infinite

34. In systematic sampling, the population is 200 and the selected sample size is 50, then the sampling interval is :

(A) 0.25

(B) 40

(C) 4

(D) 250

35. A population of 108 units is divided into three strata whose sizes are 24,36 and 48 respectively. In proportion allocation, the number of units selected from the second stratum when sample size is 18, will be :

(A) 4

(B) 6

(C) 8

(D) 9

36. The criterion, which enables us to classify variou sampling units into different strata, is termed as

(A) Inflation factor

(B) Sampling factor

(C) Sampling fraction

(D) Stratifying factor

37. If G is the grand total, T is the number of treatments and N is the total number of observations, then the correction factor is given by :

(A) $\frac{G^2}{NT}$

(B) $\frac{G}{N}$

(C) $\frac{G^2}{N}$

(D) $\frac{G^2}{T}$

38. In which of the following designs is principle of local control not used ?

(A) CRD

(B) RBD

(C) LSD

(D) BIBD

SEAL

39. In one way classification, the test statistic F is given by :

(A) $\frac{\text{Treatment sum of squares}}{\text{Error sum of squares}}$

(B) $\frac{\text{Treatment mean sum of squares}}{\text{Error mean sum of squares}}$

(C) $\frac{\text{Error mean sum of squares}}{\text{Treatment mean sum of squares}}$

(D) $\frac{\text{Raw sum of squares}}{\text{Total sum of squares}}$

40. Local control is used to :

(A) Reduce the error variance

(B) Reduce the number of replications

(C) Reduce the degree of freedom

(D) Decrease the number of plots

41. A feasible solution to an LP problem :

(A) Must satisfy all the problems constraints simultaneously

(B) Must optimize the value of the objective function

(C) Need not to satisfy all constraints, only some of them

(D) Must be a corner point of the feasible region

42. While solving LP Problem graphically, the area bounded by the constraints is known as :

(A) Infeasible region

(B) Feasible region

(C) Unbounded region

(D) Critical region

43. For maximization LP model, the simplex method is terminated when all values :

(A) $C_j - Z_j \geq 0$

(B) $C_j - Z_j = 0$

(C) $Z_j \leq 0$

(D) $C_j - Z_j \leq 0$

44. A solution which can indefinitely increase or decrease the value of the objective function is called :

(A) Degenerate solution

(B) Feasible solution

(C) Unbounded solution

(D) Optimal solution

45. The trend of annual production of a company is described by the following equation $Y_c = 18+0.6X$.

Origin 1988; X-unit = 1 year; Y-unit = annual production. The monthly trend equation is :

(A) $Y_c = 1.50 + 0.0041X$

(B) $Y_c = 0.01 + 0.05X$

(C) $Y_c = 1.80 + 0.06X$

(D) $Y_c = 1.50 + 0.05X$

46. Which of the following is an example of seasonal variation ?

(A) Recovery in business

(B) Death rate decreased due to advance in science

(C) Sudden causes by war

(D) Sale of ice cream increases during summer

47. The multiplicative time series model is :

(A) $Y = T + S + C + I$

(B) $Y = T \times S \times C \times I$

(C) $Y = a + bX$

(D) $Y = a + bX + cX^2$

48. Fisher's method of calculating the index number is based on :

(A) Geometric mean

(B) Arithmetic mean

(C) Harmonic mean

(D) Quartile deviation

49. The number of births per thousand women of child bearing age group is :

(A) Crude birth rate

(B) General fertility rate

(C) Total fertility rate

(D) Gross reproduction rate

50. Natural increase indicates :

(A) Higher mortality

(B) An excess of deaths over births

(C) Higher fertility

(D) An excess of births over deaths

51. The child bearing age in India is :

(A) 15-59 years

(B) 20-29 years

(C) 15-49 years

(D) 14-49 years

52. Mortality experience is taken into consideration when defining :

(A) Crude birth rate

(B) General fertility rate

(C) Gross reproduction rate

(D) Net reproduction rate

53. Computer is free from tiresome and boredom. We call it :

(A) Diligence

(B) Intelligence

(C) Robotics

(D) Accuracy

54. Assignable causes of variation :

(A) Are not as important as natural causes

(B) Can be identified and removed

(C) Also referred to as "chance" causes

(D) Are within the limits of a control chart

55. Central tendency of a process is monitored in :

(A) Range chart

(B) Mean chart

(C) p-chart

(D) c-chart

56. Process control is carried out :

(A) Before production

(B) After production control

(C) During production

(D) All of the above

57. Let T_1 be the most efficient estimator of θ with variance σ_1^2 and T_2 be any other estimator of the same parameter of θ with variance σ_2^2. The efficiency of T_2 is :

(A) σ_1^2/σ_2^2

(B) σ_2^2/σ_1^2

(C) σ_1/σ_2

(D) σ_2/σ_1

58. If $X_i \sim N(\mu, \sigma^2)$ where σ^2 is known, then 95% confidence interval for μ based on a sample of size n is :

(A) $\left(\overline{X} \pm 2.00^{\sigma}/\sqrt{n}\right)$

(B) $\left(\overline{X} \pm 2.58^{\sigma}/\sqrt{n}\right)$

(C) $\left(\overline{X} \pm 1.96^{\sigma}/\sqrt{n}\right)$

(D) $\left(\overline{X} \pm 1.00^{\sigma}/\sqrt{n}\right)$

59. Let (Y_1, Y_2, Y_3) be a random sample from a population having mean θ. Let $T = \frac{(2Y_1 + 3Y_2 + \lambda Y_3)}{9}$. If T is given to be an unbiased estimator of θ, then the value of λ is :

(A) 1

(B) 2

(C) 3

(D) 4

60. A statistic is called sufficient for a parameter if it :

(A) Possesses all three criteria of unbiasedness, consistency and efficiency

(B) Gives among all the statistics, the maximum of the information about parameter

(C) Gives all information about the parameter that is contained in the sample

(D) Gives more information about the parameter than that is contained in the sample

Sr. No. 080

ENTRANCE TEST-2020

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

STATISTICS

Question Booklet Series **D**

Total Questions : 60
Time Allowed : 70 Minutes

Roll No. :

Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be **'Negative Marking'** for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer Sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

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1. Non negativity condition is an important component of LP model because :
   (A) Variable value should remain under the control of decision maker
   (B) Value of the variable make sense and correspond to real world problems
   (C) Variables are interrelated in terms of limited resources
   (D) None of the above
2. While solving a LP model graphically, the area bounded by the constraints is called :
   (A) Feasible region
   (B) Infeasible region
   (C) Unbounded solution
   (D) None of the above
3. The solution to a transportation problem with m supplies and n destinations is feasible if number of positive allocation are :
   (A) $m + n$
   (B) $m \times n$
   (C) $m + n - 1$
   (D) $m + n + 1$
4. An assignment problem can be solved by :
   (A) Simplex method
   (B) Transportation method
   (C) Both (A) and (B)
   (D) None of the above
5. The component of a time series attached to long term variations is termed as :
   (A) Cyclic variation
   (B) Secular trend
   (C) Irregular variation
   (D) All the above
6. A linear trend shows the business movement of a time series towards :
   (A) Growth
   (B) Decline
   (C) Stagnation
   (D) All the above
7. Paasche's index number was invented in the year :
   (A) 1871
   (B) 1874
   (C) 1901
   (D) 1918
8. Laspeyre's index formula uses the weights of the :
   (A) Base year
   (B) Current year
   (C) Average of weights of a number of years
   (D) None of the above
9. In India, the collection of vital statistics started for the first time in :
   (A) 1720
   (B) 1886
   (C) 1946
   (D) 1969
10. In post-independence India, the Registration of Births and Deaths Act was passed in :
   (A) 1948
   (B) 1959
   (C) 1969
   (D) 1979
11. Total fertility rate provides the basis for :
   (A) The expected family size
   (B) Population projection
   (C) Population increase in a desired period
   (D) All the above
12. The probability of dying of a person of age between x and x+1 year is known as :
   (A) Age specific death rate
   (B) Central mortality rate
   (C) Infant mortality rate
   (D) None of the above
13. Chance variation in respect of quality control of a product is :
   (A) Tolerable
   (B) Not effecting the quality of a product
   (C) Uncontrollable
   (D) All the above
14. The number of mistakes committed by a mechanic in 20 sample of assembled T.V. are 25. The lower control limit for c chart for the given data is :
   (A) 10
   (B) 1.25
   (C) 0.8
   (D) None

15. A factory produces 300 articles per day. After inspecting 3000 articles on 30 consecutive days, 270 articles were non-conforming to the specification. The upper control limit for p-chart is :
(A) 0.14
(B) 0.24
(C) 1.14
(D) None

16. The Schewhart control charts are meant :
(A) To detect whether the process is under statistical quality control
(B) To find the assignable cause
(C) To reflect the selection of sample
(D) All the above

17. Let $x_1, x_2, \ldots\ldots, x_n$ be a random sample from a Bernoulli population $p^x q^{n-x}$ for $x = 0,1$ and $0<p<1$, $q=1-p$. A sufficient statistics for p is :
(A) $\prod_1^n x_i$
(B) $\sum_1^n x_i$
(C) Max $(x_1, x_2, \ldots x_n)$
(D) None

18. Factorization theory for sufficiency is known as :
(A) Cramer Rao theorem
(B) Rao Blackwell theorem
(C) Fisher Neyman theorem
(D) Chapman Robins theorem

19. Let $x_1, x_2, \ldots\ldots, x_n$ be a random sample of size n from a population having pdf $f(x, \theta)$, where

$$f(x,\theta), = \frac{1}{\theta} e^{-\frac{x}{\theta}} \quad , 0<x<\infty, 0<\theta<\infty$$

The unbiased estimator of θ^2 is :
(A) $\frac{nx}{n+1}$
(B) $\frac{\bar{x}^2}{n+1}$
(C) $\frac{n\bar{x}^2}{n+1}$
(D) None

20. Let $x_1, x_2, \ldots\ldots, x_n$ be a random sample of n observations from a population with pdf

$$f(x,\theta), = \theta x^{\theta-1} \quad , 0<x<1, 0<\theta<\infty$$

The ML estimator of θ is :
(A) $-\frac{n}{\sum \log x_i}$
(B) $-\frac{nx}{\sum \log x_i}$
(C) $-\frac{nx^2}{\sum \log x_i}$
(D) None

21. The algebraic sum of the deviations of a set of n values from their arithmetic mean is :
(A) 0
(B) 1
(C) n
(D) ∞

22. 10 is the mean of a set of 7 observations and 5 is the mean of a set of 3 observations. The mean of the combine set is :
(A) 6.5
(B) 7.5
(C) 8.5
(D) 15

23. The first of the two samples has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$. The standard deviation of the second group is :
(A) 4
(B) 8
(C) 12
(D) 16

24. If mode is ill defined, then Karl-Pearson coefficient of skewness for a moderately asymmetrical data is given by :

(A) $\frac{(\text{Mean} - \text{Median})}{3\sigma}$

(B) $\frac{2\text{Mean} - 3\text{Median}}{\sigma}$

(C) $\frac{3\text{Mean} - 2\text{Median}}{\sigma}$

(D) $\frac{3(\text{Mean} - \text{Median})}{\sigma}$

25. The variable X and Y are connected by the equation $AX + BY + C = 0$. If A and B are of opposite sign, then correlation coefficient between X and Y is :

(A) -1
(B) 0
(C) 1
(D) None

26. If r is the correlation coefficient in sample of n pairs of observations, then probable error of correlation coefficient is :

(A) $\frac{1-r^2}{\sqrt{n}}$

(B) $0.6745\frac{1-r^2}{n}$

(C) $0.6745\sqrt{\frac{1-r^2}{n}}$

(D) $0.6745\frac{1-r^2}{\sqrt{n}}$

27. The correlation coefficient for the regression equations $8X - 10Y + 66 = 0$ and $40X - 18Y - 214 = 0$ is :

(A) ± 0.3
(B) ± 0.6
(C) ± 0.8
(D) None

28. The Spearman's rank correlation coefficient will be maximum when :

(A) Each of the deviations is minimum
(B) Each of the deviations is maximum
(C) Both (A) and (B)
(D) Neither (A) nor (B)

29. Two unbiased coins are tossed. The probability of getting at least one head is :

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) None

30. The first of the three urns contains 7 white and 10 black balls, second contains 5 white and 12 black balls and the third contains 17 white. A person chooses an urn at random and draws a ball. The ball is white. The probability that the ball comes from the third urn is :

(A) $\frac{15}{87}$

(B) $\frac{21}{87}$

(C) $\frac{51}{87}$

(D) None

31. If $B \subset A$, then $P(A \cap \overline{B})$ is equal to :

(A) $P(B) - P(A)$
(B) $P(A) - P(B)$
(C) $P(A) \cdot P(B)$
(D) None

32. For any two events A and B, $P(\overline{A} \cap B)$ is given by :

(A) $P(A) - P(A \cup B)$
(B) $P(A) - P(A \cap B)$
(C) $P(B) - P(A \cup B)$
(D) $P(B) - P(A \cap B)$

33. Let X be a continuous random variable with p.d.f. :

$f(x) = ax ; \quad 0 \le x \le 1$

$= a ; \quad 1 \le x \le 2$

$= -ax + 3a; \quad 2 \le x \le 3$

$= 0 \quad$ elsewhere

The value of the constant a is :

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

34. Let the random variable X assume the value r with the probability law $P(X=r) = pq^{r-1}$; $r = 1, 2, \ldots$, then the variance of the variable is :

(A) $\frac{p}{q^2}$

(B) $\frac{q}{p^2}$

(C) $\frac{1}{pq}$

(D) None

35. The continuous random variable X has the pdf :

$f(x) = \frac{2}{x^3} \quad 1 < x < \alpha$

$= 0 \quad$ otherwise

Then E(2X – 1) is equal to :

(A) 1

(B) 2

(C) 3

(D) 4

36. In hyper-geometric distribution H.G(N, α, n) if $N \to \infty, \frac{\alpha}{N} \to p$, then the hyper-geometric distribution reduces to :

(A) Poisson distribution

(B) Binomial distribution

(C) Geometric distribution

(D) None

37. The distribution function of a continuous uniform distribution of a variable X lying in the interval (0, 1) is :

(A) X

(B) $\frac{1}{X}$

(C) 1

(D) X^2

38. If $X \sim N(8, 1)$, the probability density function of the variable X is :

(A) $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(X-8)^2}$

(B) $\frac{1}{8\sqrt{2\pi}} e^{-\frac{1}{2}(X-1)^2}$

(C) $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-1}{8}\right)^2}$

(D) None

39. The probability density function of beta distribution of first kind with parameters $\alpha, \beta > 0$ is :

(A) $\frac{1}{B(\alpha,\beta)} x^{\alpha-\beta}(1-x)^{\alpha-1}; 0 < x < 1$

(B) $\frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}; 0 < x < 1$

(C) $\frac{1}{B(\alpha,\beta)} x^{\beta-1}(1-x)^{\beta-\alpha}; 0 < x < 1$

(D) None

40. If $X \sim Exp(1)$, the probability density function of X is :

(A) $1/e^x$, for $x > 0$

(B) e^{x^2}, for $x > 0$

(C) e^x, for $x > 0$

(D) e^{-x}, for $x > 0$

41. The probability limit for the observed proportion of successes are :

(A) $Q \pm 3\sqrt{\frac{PQ}{n}}$

(B) $P \pm 3\sqrt{\frac{PQ}{n}}$

(C) $P \pm 3\sqrt{\frac{PQ}{N}}$

(D) None

42. Degree of freedom is related to :
(A) Hypothesis under test
(B) No. of observations in a set
(C) No. of independent observations in a set
(D) All the above

43. A test is one sided or two sided depends on :
(A) Null hypothesis
(B) Composite hypothesis
(C) Simple hypothesis
(D) Alternative hypothesis

44. Area of critical region depends on :
(A) Value of the statistics
(B) No. of observations
(C) Size of type I error
(D) Size of type II error

45. In a sample of 8 observations the sum of squares of deviations of the sample values from the sample mean is 84.4 and in the other sample of 10 observations it is 102.6, the value of the test statistics is :
(A) 0.8
(B) 0.819
(C) 1.028
(D) 1.057

46. A population is distributed as N(5.2, 10.24). A sample of 576 item has a mean 4.7. The value of the test statistics to test $H_0 : \mu = 5.2$ is :
(A) –3.75
(B) 1.73
(C) 3.75
(D) None

47. Test of hypothesis $H_0 : \mu = 5$ vs $H_0 : \mu >= 5$ leads to :
(A) One side left tailed test
(B) One side right tailed test
(C) Two tailed test
(D) None of the above

48. A random sample of 27 pairs of observations from a normal population gave a correlation coefficient of 0.6. The value of the test statistics is :
(A) 1.75
(B) 2.75
(C) 3.75
(D) None

49. Which of the following is non-probability sampling ?
(A) Quota sampling
(B) Judgment sampling
(C) Both (A) and (B)
(D) Neither (A) nor (B)

50. The estimate of population total Y is :
(A) $\bar{y}$
(B) $n\bar{y}$
(C) $N\bar{y}$
(D) $N\bar{Y}$

51. An estimator of standard error of $\bar{y}$ is :
(A) $\frac{N-n}{nN}S$
(B) $\frac{N-n}{N}S$
(C) $\frac{N-n}{nN}S^2$
(D) None

52. A combination of a sampling design and an estimator is called :
(A) Sampling strategy
(B) Sampling error
(C) Sampling frame
(D) None

53. Standard error of estimate of the population total in stratified sampling is given by :
(A) $N\sqrt{\frac{\sum W_h^2(1-f_h)s_h^2}{n_h}}$
(B) $N\sqrt{\frac{\sum W_h^2(1-f_h)S_h^2}{n_h}}$
(C) $N\sqrt{\frac{\sum W_h^2(1-f_h)S_h^2}{N_h}}$
(D) All the above

54. The reduction in variance from proportional allocation to optimum allocation is caused by the variation between :
(A) s_h^2
(B) S_h^2
(C) S^2
(D) None

55. If fpc is ignored in proportional allocation the $V(\bar{y}_{st})_{prop}$ is given by :
(A) $\sum_h \frac{W_h S_h^2}{n}$
(B) $\sum_h \frac{W_h S_h^2}{N}$
(C) $\sum_h \frac{W_h S_h^2}{n}$
(D) None

56. The efficiency of the systematic sampling compared to SRS depends on the :
(A) Intra class correlation coefficient
(B) Interclass correlation coefficient
(C) Both (A) and (B)
(D) Neither (A) nor (B)

57. Randomization in an experiment helps to eliminate :
(A) Systematic influence
(B) Human biases
(C) Dependence among observations
(D) None of the above

58. Errors in a statistical model are always taken to be :
(A) Independent
(B) Distributed as $N(0, \sigma^2)$
(C) Both (A) and (B)
(D) Neither (A) nor (B)

59. In a completely randomized design with t treatment and n experimental units, error degree of freedom is equal to :
(A) t–n
(B) n–t–1
(C) n–t+1
(D) n–t

60. A randomized block design has :
(A) One way classification
(B) Two way classification
(C) Three way classification
(D) No Classification

1. For a finite population of size N = 500, the population proportion is known to be 0.85. How large a sample should be taken in order to estimate P with margin of error 0.05 with confidence coefficient 0.95 ?

(A) 196

(B) 174

(C) 141

(D) 125

2. For srswor (N, n), an unbiased variance estimator of Y is :

(A) $\frac{N-n}{nN}S^2$

(B) $\frac{N(N-n)}{n}S^2$

(C) $\frac{nN}{N(N-1)}S^2$

(D) None of the above

3. Sample variance of the h^{th} stratum is given by :

(A) $\sum_{i=1}^{n_h}\frac{(y_{hi}-\bar{y}_h)^2}{n_h-1}$

(B) $\sum_{i=1}^{n_h}\frac{(Y_{hi}-\bar{Y}_h)^2}{n_h-1}$

(C) $\sum_{i=1}^{N_h}\frac{(Y_{hi}-\bar{Y}_h)^2}{N_h-1}$

(D) None

4. Which one is true in stratified sampling ?

(A) Variance within stratum is lesser than the variance between stratum.

(B) Variance within stratum is higher than the variance between stratum.

(C) Both (A) and (B)

(D) Neither (A) nor (B)

5. The mean of a systematic sample is more precise than the mean of a simple random sample if and only if:

(A) $s^2 > s^2_{wsy}$

(B) $s^2 < s^2_{wsy}$

(C) $\rho_{wsy} > 1$

(D) None

6. The variance of the systematic sampling is zero, when the intra class correlation coefficient is :

(A) $\frac{1}{1-n}$

(B) $\frac{1}{N-1}$

(C) $\frac{n}{N-1}$

(D) None

7. Randomization in an experiment helps to eliminate :

(A) Systematic influence

(B) Human biases

(C) Dependence among observation

(D) All the above

8. Number of replications in an experiment is based on :

(A) The precision required

(B) Experimental material available

(C) Heterogeneity of the experimental material

(D) All the above

9. Local control in experimental design is meant to :

(A) Increase the efficiency of the design

(B) Reduce experimental error

(C) To form homogeneous blocks

(D) All the above

⊠⊠⊠

10. A Latin square design possesses :
    (A) One way classification
    (B) Two way classification
    (C) Three way classification
    (D) None of the above
11. In linear programming problem :
    (A) Objective function, constraints and variables are all linear
    (B) Only objective function to be linear
    (C) Only constraints to be linear
    (D) Only variables to be linear
12. The variable to be determined in LPP are always :
    (A) Positive only
    (B) Negative only
    (C) Non-negative
    (D) Non-positive
13. The set of basic optimal solution to an LPP is :
    (A) Finite
    (B) Convex
    (C) Either singleton or infinite
    (D) None of the above
14. The solution of transportation problem with m rows and n columns is feasible if number of positive allocation are :
    (A) $m + n$
    (B) $m \times n$
    (C) $m + n - 1$
    (D) $m + n + 1$
15. Most preferred type of average for index number is :
    (A) Arithmetic mean
    (B) Geometric mean
    (C) Harmonic mean
    (D) None of the above
16. Laspeyre's index formula uses the weights of the :
    (A) Base year
    (B) Current year
    (C) Average of the weights of a number of years
    (D) None of the above
17. If the index number is independent of the units of measurements, then it satisfies :
    (A) Time reversal test
    (B) Factor reversal test
    (C) Unit test
    (D) All the above
18. The conditions for the time reversal test to hold good with usual notation is :
    (A) $P_{01} \times P_{10} = 1$
    (B) $P_{10} \times P_{01} = 0$
    (C) $P_{01}/P_{10} = 1$
    (D) $P_{01} + P_{10} = 1$
19. Sampling registration system of births and deaths came into operation in rural area in the year :
    (A) 1967
    (B) 1968
    (C) 1969
    (D) None of the above
20. The ratio of births to the total deaths in a year is called :
    (A) Survival rate
    (B) Total fertility index
    (C) Vital index
    (D) Population death rate
21. If P_1 and P_2 are the population at an interval of ten years, the population just after five years will be :
    (A) $(P_1 + P_2)/2$
    (B) $\sqrt{P_1 \times P_2}$
    (C) $\frac{1}{2}\left(\frac{1}{P_1} + \frac{1}{P_2}\right)$
    (D) $\sqrt{P_1 + P_2}$
22. Fertility rate mainly depends on :
    (A) Total female population
    (B) Total population
    (C) Female population of child bearing age
    (D) Number of newly born babies

23. Main tools of statistical quality control are :
(A) Schwartz Charts
(B) Acceptance sampling plan
(C) Both (A) and (B)
(D) Neither (A) nor (B)

24. Variation in the items produced in a factory may be due to :
(A) Chance factor
(B) Assignable causes
(C) Both (A) and (B)
(D) None of the above

25. The relation between expected value of R and S.D. σ with usual constant factors is :
(A) $E(R) = d_1\sigma$
(B) $E(R) = d_2\sigma$
(C) $E(R) = D_1\sigma$
(D) $E(R) = D_2\sigma$

26. Variation due to assignable causes in the product occurs due to :
(A) Faulty process
(B) Carelessness of operators
(C) Poor quality of raw material
(D) All the above

27. The variance of maximum likelihood estimate for the parameters λ of a Poisson distribution on the basis of sample size n is :
(A) λ
(B) λ/n
(C) n/λ
(D) λ/n^2

28. Let $x_1, x_2, ..., x_n$ be a random sample from a population with pdf :
$f(x, \theta) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0.$
Then $t = \Pi x_i$ is :
(A) Sufficient estimator for θ
(B) Sufficient estimator for $n\theta$
(C) Sufficient estimator for n/θ
(D) None

29. If T_1 is a MVU for θ and T_2 is any other unbiased estimator of θ with efficiency e, then the correlation coefficient between T_1 and T_2 is given by :
(A) $\sqrt{e}$
(B) e
(C) 1/e
(D) None

30. If an estimator T_n of population parameter θ converges in probability to θ as $n \to \infty$ is said to be :
(A) Sufficient
(B) Efficient
(C) Consistent
(D) Unbiased

31. Geometric mean is better than other means when :
(A) The data are positive as well as negative
(B) The data are in ratios or percentages
(C) The data are binary
(D) The data are on interval

32. The arithmetic mean of two numbers is 6.5 and their G.M. is 6, the two numbers are :
(A) 9, 6
(B) 9, 5
(C) 7, 6
(D) 4, 9

33. If A, G and H denote the arithmetic mean, geometric mean and harmonic mean respectively, then :
(A) $A \leq G \leq H$
(B) $G^2 = AH$
(C) $G = AH$
(D) None

34. If each value of a series is multiplied by 10, the coefficient of variation will be increased by :
(A) 15%
(B) 10%
(C) 5%
(D) None

35. The standard error of the correlation coefficient is given by :

(A) $\frac{1-r}{\sqrt{n}}$

(B) $\frac{1-r}{n}$

(C) $\frac{1-r^2}{\sqrt{n}}$

(D) $\frac{1-r^2}{n}$

36. Regression co-efficient is independent of :

(A) Origin
(B) Scale
(C) Both (A) and (B)
(D) Neither (A) nor (B)

37. Given two lines of regression as $3X - 4Y + 8 = 0$ and $4X - 3Y = 1$, the means of X and Y are :

(A) 4, 5
(B) 3, 4
(C) $\frac{4}{3}, \frac{5}{4}$
(D) None

38. If $\rho = 0$, the lines of regression are :

(A) Coincident
(B) Parallel
(C) Perpendicular to each other
(D) None of the above

39. In a single throw of two dice the probability of getting a total different from 8 is :

(A) $\frac{5}{36}$

(B) $\frac{12}{36}$

(C) $\frac{25}{36}$

(D) $\frac{31}{36}$

40. One ticket is drawn at random from a bag containing 30 tickets numbered from 1 to 30, the probability that it is a multiple of 5 or 7 is :

(A) $\frac{1}{3}$

(B) $\frac{1}{5}$

(C) $\frac{1}{7}$

(D) $\frac{2}{5}$

41. Two digits are selected at random from the digits 1 to 9. If the sum is even, the probability that both are odd is :

(A) $\frac{1}{6}$

(B) $\frac{5}{8}$

(C) $\frac{5}{18}$

(D) $\frac{8}{18}$

42. If A and B are two events, the probability of occurrence of either A or B is given as :

(A) $P(A) + P(B)$
(B) $P(A \cup B)$
(C) $P(A \cap B)$
(D) $P(A) . P(B)$

43. The variance of three tosses of a coin is :

(A) $\frac{3}{2}$

(B) $\frac{9}{4}$

(C) $\frac{3}{4}$

(D) 3

⊠⊠⊠

44. A discrete random variable can take all possible integer values from 1 to k each with a probability $\frac{1}{k}$, the mean of the variable is :

(A) $\frac{k+1}{2}$

(B) $\frac{(k+1)(2k+1)}{6}$

(C) $\frac{2k+1}{6}$

(D) $\frac{k^2-1}{12}$

45. A dice is thrown at random, the expectation of the number is :

(A) $\frac{1}{2}$

(B) $\frac{1}{6}$

(C) $\frac{2}{7}$

(D) $\frac{7}{2}$

46. The value of p for a binomial random variable X, if $n = 6$ and $9\,P(X=4) = P(X=2)$ is :

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{2}{3}$

47. The mean of distribution $f(x) = \frac{1}{\Gamma(\alpha)\,\beta^{\alpha}} x^{\alpha-1} e^{x/\beta}$: $0 < x < \infty$, is :

(A) α/β

(B) $\alpha\beta$

(C) $\alpha\beta^2$

(D) None of the above

48. If we put $\alpha = 1$ in the pdf $f(x) = \frac{1}{\Gamma(\alpha)\,\beta^{\alpha}} x^{\alpha-1} e^{x/\beta}$: $0 < x < \infty$, it reduces to :

(A) Normal distribution

(B) Exponential distribution

(C) Beta distribution of Ist kind

(D) None of the above

49. The mean of the distribution :

$$f(x) = \frac{\Gamma\alpha + \beta}{\Gamma\alpha\,\Gamma\beta} x^{\alpha} (1-x)^{\beta};\ 0 < x < 1 :$$

(A) α/β

(B) $\frac{\alpha+\beta}{\alpha}$

(C) $\frac{\alpha}{\alpha+\beta}$

(D) $\frac{\beta}{\alpha+\beta}$

50. The mgf of the random variable X having pdf $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, -\infty < x < \infty$ is :

(A) $e^{t^2/2}$

(B) e^{t^2}

(C) $2e^{t^2/2}$

(D) None

⊠⊠⊠

Degree of freedom is related to :

(A) Number of observations in a set

(B) Hypothesis under test

(C) Number of independent observations in a set

(D) None of the above

Area of the critical region depends on :

(A) Size of type I error

(B) Size of type II error

(C) Value of Statistics

(D) Number of observations

Level of significance is the probability of :

(A) Type I error

(B) Type II error

(C) Both (A) and (B)

(D) Neither (A) nor (B)

4. Testing $H_0 : 100$ vs $H_1 \neq 100$ leads to :

(A) One sided upper tailed test

(B) One sided lower tailed test

(C) Two tailed test

(D) None of the above

55. The degree of freedom for chi-square in case of contingency table of order (4×3) are :

(A) 12

(B) 9

(C) 8

(D) 6

56. Given the sample statistics, $n_1 = 400$, $\bar{x}_1 = 24.50$, $s_1 = 2.5$ $n_2 = 500$, $\bar{x}_2 = 20.00$, $s_2 = 2.0$. The value of the test statistics to test $H_0 : \mu_1 = \mu_2$, when $\sigma_1^2 = \sigma_2^2$ is :

(A) 44.47

(B) 30.00

(C) 8.97

(D) None

57. Given the sample statistics as $n = 51$, $\sigma = 8$ and $s = 10$, the value of the test statistic is :

(A) 79.69

(B) 64.27

(C) 59.23

(D) None of the above

58. A random sample of 27 pairs of observations from a normal population gives a correlation coefficient 0.42. The value of the test statistics is :

(A) 2.49

(B) 2.31

(C) 2.12

(D) 1.92

59. A combination of sampling design and an estimator is called :

(A) Sampling strategy

(B) Sampling frame

(C) Both (A) and (B)

(D) Neither (A) nor (B)

60. In srswr (N, n), an unbiased estimator of Y is :

(A) $N\bar{Y}$

(B) $N\bar{y}$

(C) $n\bar{y}$

(D) None of the above

Option—(i) : Statistics

1. If the two observations are 5 and –5, then their harmonic mean is :

   (A) 5

   (B) 0

   (C) –5

   (D) None

2. In a class test, 20 students out of 40 passed with mean marks 7.00 and the overall average of class was 6.00. The average marks of students who failed were :

   (A) 2.5

   (B) 3.5

   (C) 5.00

   (D) 6.00

3. The variance of first n natural number is :

   (A) $\frac{(n^2-1)}{12}$

   (B) $\frac{(n^2+1)}{12}$

   (C) $\frac{(2n^2-1)}{8}$

   (D) $\frac{(n+1)^2}{4}$

4. For a negatively skewed frequency distribution curve, the third central moment is :

   (A) $\mu_3 > 0$

   (B) $\mu_3 < 0$

   (C) $\mu_3 = 0$

   (D) None of the above

5. The line of regression intersect at the point :

   (A) (0, 0)

   (B) (1, 1)

   (C) (X, Y)

   (D) $(\overline{X}, \overline{Y})$

6. If r = 0 the angle θ between the two lines of regression is :

   (A) $\theta = \pi$

   (B) $\theta = \frac{\pi}{2}$

   (C) $\theta = \frac{\pi}{4}$

   (D) $\theta = 0$

7. The probability of error of correlation coefficient is given by :

   (A) $0.6745 \times \frac{(1-r^2)}{n}$

   (B) $0.6745 \times \frac{(1-r)^2}{n}$

   (C) $0.6745 \times \frac{(1-r)^2}{\sqrt{n}}$

   (D) $0.6745 \times \frac{(1-r^2)}{\sqrt{n}}$

8. Given the two lines of regression as $8X - 10Y + 66 = 0$ and $40X - 18Y = 214$ the mean of X and Y are :

   (A) $\overline{X} = 13, \overline{Y} = 17$

   (B) $\overline{X} = 8, \overline{Y} = 40$

   (C) $\overline{X} = 40, \overline{Y} = 18$

   (D) $\overline{X} = 8, \overline{Y} = -10$

9. Twelve balls are distributed at random among three boxes. The probability that the first box will contain three balls is :

(A) 0.212

(B) 0.235

(C) 0.245

(D) 0.52

10. If A and B are two events, the probability of event A occur and B does not occur is :

(A) $P(A \cap \overline{B})$

(B) $P(\overline{A} \cap B)$

(C) $P(A \cup \overline{B})$

(D) $P(\overline{A} \cup B)$

11. For any three events A, B and C defined on the sample space S, such that $B \subset C$ and $P(A) > 0$, then :

(A) $P(B|A) < P(C|A)$

(B) $P(B|A) > P(C|A)$

(C) $P(B|A) \leq P(C|A)$

(D) $P(B|A) \geq P(C|A)$

12. If two dice are thrown, the probability that the sum is neither 7 nor 11 is :

(A) $\frac{2}{3}$

(B) $\frac{1}{18}$

(C) $\frac{1}{6}$

(D) $\frac{18}{36}$

13. A continuous random variable X has a probability density function $f(x) = 3x^2; 0 \leq x \leq 1$.

If $P(X \leq a) = P(X > a)$, then a is equal to :

(A) $\left(\frac{1}{2}\right)^{\frac{1}{3}}$

(B) $\left(\frac{1}{3}\right)^{\frac{1}{2}}$

(C) $\left(\frac{1}{6}\right)$

(D) None of the above

14. The mean and variance of a binomial distribution are 8 and 4 respectively.

Then $P(X = 1)$ is equal to :

(A) $\frac{1}{2^4}$

(B) $\frac{1}{2^6}$

(C) $\frac{1}{2^8}$

(D) $\frac{1}{2^{12}}$

15. In a hyper-geometric distribution (N, M, n), if $N \to \infty$, $\frac{M}{N} \to p$, then the Hyper-geometric distribution reduces to :

(A) Geometric distribution

(B) Poisson distribution

(C) Binomial distribution

(D) None of the above

16. The moment generating function of the geometric distribution with usual notation is :

(A) $\frac{p}{1 - qe^t}$

(B) $\frac{q}{1 - pe^t}$

(C) $\frac{p}{1 + qe^t}$

(D) $\frac{q}{1 + pe^t}$

17. Subway trains on a certain line run every half hour between mid-night and six in the morning. The probability that a man entering station at a random time during this period is :

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{5}$

(D) $\frac{1}{4}$

18. A linear combination of independent normal variate is :

(A) Uniform variate

(B) Normal variate

(C) Gamma variate

(D) Exponential variate

19. For large value of λ, the gamma distribution $\gamma(\lambda)$ tends to :

(A) Exponential distribution

(B) Beta distribution of first kind

(C) Uniform distribution

(D) Normal distribution

20. The mean of the beta distribution of first kind $B(\mu, \nu)$ with usual notation is :

(A) $\frac{\mu}{\mu + \nu}$

(B) $\frac{\nu}{\mu + \nu}$

(C) $\frac{\mu}{\mu - \nu}$

(D) $\frac{\nu}{\mu - \nu}$

21. The probability that a random value of the vector $X = (x_1, x_2, ..., x_n)$ belongs to the critical region is known as :

(A) Level of significance

(B) Size of type I error

(C) Both (A) and (B)

(D) Neither (A) nor (B)

22. A random sample of 500 apples was taken from a large consignment and 65 were found to be bad. The S.E. of proportion is :

(A) 0.015

(B) 0.012

(C) 0.5

(D) None of the above

23. In two large populations, there are 30 and 25 percent respectively of fair haired people. The value of the test statistics for the difference of proportion is :

(A) 2.46

(B) 2.52

(C) 2.56

(D) 1.74

24. A sample of 900 members has a mean 3.4 cm and s.d. 2.61 cm. The sample has been drawn from a large population of mean 3.25 cm and s.d. 2.61 cm. 95% confidence limit for the population mean is :

(A) (3.5705 and 3.2295)

(B) (3.6251 and 3.2295)

(C) (3.5705 and 3.9225)

(D) (3.6251 and 3.9225)

25. In a sample of 8 observations, the sum of the squares of deviation of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102.6. The value of the test statistic is :

(A) 1.024

(B) 1.057

(C) 1.145

(D) None of the above

26. A random sample of 27 pairs of observations from a normal population gave a correlation coefficient of 0.6. The value of the test statistic is :

(A) 3.75

(B) 3.54

(C) 3.25

(D) 2.93

27. Degree of freedom for χ^2 in case of contingency table of order 4×3 are :

(A) 12

(B) 9

(C) 8

(D) 6

28. Given the sample statistics

$$n_1 = 8, \quad \bar{x}_1 = 1234, \quad s_1 = 36$$

$$n_2 = 7, \quad \bar{x}_2 = 1036, \quad s_2 = 40$$

the value of the test statistic to test $H_0 : \mu_1 = \mu_2$ is :

(A) 8.37

(B) 8.65

(C) 9.37

(D) 9.45

29. For a simple random sampling without replacement srswor (N, n), the probability of two specified units being selected at any given draw is :

(A) $\frac{n(n-1)}{N(N-1)}$

(B) $\frac{N}{n(n-1)}$

(C) $\frac{1}{N(N-1)}$

(D) $\frac{1}{n(n-1)}$

30. For an srswor (N, n), the probability that any two specified units are included in the sample is :

(A) $\frac{n(n-1)}{N(N-1)}$

(B) $\frac{n-2}{(N-2)}$

(C) $\frac{1}{N(N-1)}$

(D) $\frac{1}{n(n-1)}$

31. The probability of getting a sample of size n from simple random sampling with replacement srswr (N, n) is :

(A) $\frac{n}{N}$

(B) $\frac{1}{n^N}$

(C) $\frac{1}{N^n}$

(D) $\frac{1}{n}$

32. A population consisting of the results of the conceptually repeated trial is known as :

(A) Real population

(B) Finite population

(C) Infinite population

(D) None of the above

33. Under proportional allocation, the size of the sample from each stratum depends on :

(A) Population size

(B) Total sample size

(C) Size of the stratum

(D) All the above

34. In stratified random sampling with srswor in each stratum, an unbiased estimator of $V(\bar{y}_{st})$ ignoring fpc is :

(A) $\sum \frac{W_h^2 s_h^2}{n_h}$

(B) $\sum \frac{W_h^2 s_h^2}{N_h}$

(C) $\sum \frac{W_h^2}{n_h} S_h^2$

(D) $\sum \frac{W_h^2}{N_h} S_h^2$

35. The mean of a systematic sample is more precise than the mean of a simple random sample if and only if:

(A) $S^2 < S_{wsy}^2$

(B) $S^2 > S_{wsy}^2$

(C) $\rho_{wsy} > 1$

(D) None of the above

36. Systematic sampling mean :

(A) Selection of n continuous units

(B) Selection of n largest units

(C) Selection of n middle units in a sequence

(D) Selection of n units situated at equal distance

37. Randomization is a process in which the treatments are allocated to the experimental units :

(A) in a sequence

(B) with equal probability

(C) at the will of the investigator

(D) all the above

38. Local control is a device to maintain :

(A) Homogeneity among blocks

(B) Homogeneity within blocks

(C) Both (A) and (B)

(D) Neither (A) nor (B)

39. Randomized block design is a :

(A) Three restrictional design

(B) Two restrictional design

(C) One restrictional design

(D) No classification

40. In a completely randomized design with t treatments and n experimental units, error degree of freedom is equal to :

(A) $n - t$

(B) $n - t - 1$

(C) $n - t + 1$

(D) $t - n$

41. Constraints in an LP model represents :

(A) Limitation

(B) Requirements

(C) Balancing limitation and requirements

(D) All the above

42. The distinguishing feature of an LP model is :

(A) Relation among all variables is linear

(B) It has single objective function and constraints

(C) Value of decision variables is non-negative

(D) All the above

43. Which additional variables are added to convert the LPP in the standard form ?

(A) Slack variables

(B) Surplus variables

(C) Artificial variables

(D) All the above

44. The solution of a transportation problem with m rows and n columns is feasible if number of positive allocations are :

(A) $m + n$

(B) $m \times n$

(C) $m + n - 1$

(D) $m + n + 1$

45. If Laspeyre's price index is 324 and Paasche's price index is 144, then Fisher's ideal index is :

(A) 180

(B) 216

(C) 234

(D) None of the above

46. Which index satisfies factor reversal test ?

(A) Paasche's index

(B) Laspeyre's index

(C) Fisher's ideal index

(D) Walsh's price index

47. The time series analysis helps :

(A) to compare the two or more series

(B) to know the behaviour of business

(C) to make prediction

(D) all the above

48. The best method for finding out seasonal variation is :

(A) Simple average method

(B) Ratio to moving average method

(C) Ratio to trend method

(D) None of the above

49. Vital statistics is mainly concerned with :

(A) Births

(B) Deaths

(C) Marriages

(D) All the above

50. Vital rates are customarily expressed as :

(A) Percentages

(B) Per thousand

(C) Per million

(D) Per trillion

51. The death rate of babies under one month is known as :

(A) Infant mortality rate

(B) Maternal mortality rate

(C) Neonatal mortality rate

(D) Foetal death rate

52. A life table is most utilized by :

(A) Life insurance companies

(B) General insurance companies

(C) Employment exchanges

(D) All the above

53. Main tools of statistical quality control are :

(A) Shewhart chart

(B) Acceptance sampling plans

(C) Both (A) and (B)

(D) Neither (A) nor (B)

54. The probability of accepting a lot with fraction defective P_t is known as :

(A) Consumer's risk

(B) Producer's risk

(C) Type I error

(D) None of the above

55. Control chart in statistical quality control are meant for :

(A) Describing the pattern of variation

(B) Checking whether the variability in the product is within the tolerance limit or not

(C) Uncovering whether the variability in the product is due to assignable causes or not

(D) All the above

56. R-charts are preferable over σ-charts because :

(A) R and S.D. fluctuate together in case of small sample

(B) R is easily calculable

(C) R-charts are economical

(D) All the above

57. If an estimator θ_n of population parameter θ converges in probability to θ as n tends to infinity is said to be :

(A) an unbiased estimator

(B) a consistent estimator

(C) both (A) and (B)

(D) neither (A) nor (B)

58. If $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, 1)$, then $t = \frac{1}{n}\sum x_i^2$ is an unbiased estimator of :

(A) σ^2

(B) $\sigma^2 + 1$

(C) μ^2

(D) $\mu^2 + 1$

59. The MLE are :

(A) Consistent

(B) Sufficient

(C) Both (A) and (B)

(D) Neither (A) nor (B)

60. If $x_1, x_2, ..., x_n$ be a random sample from $N(0, \theta^2)$, then the maximum likelihood estimator for θ is :

(A) $\sum_i^n x_i / n$

(B) $\sum_i^n x_i^2 / n$

(C) $\sqrt{\sum_i^n x_i^2 / n}$

(D) $\sqrt{\sum_i^n x_i^2} / n$

Option—(ii) : Mathematics

1. Which of the following is not true for the function

$$f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, \; x \neq 0$$

$$= 0, \; x = 0 \; ?$$

(A) The left hand limit of f(x) at x = 0 is –1
(B) The right hand limit of f(x) at x = 0 is 1
(C) f(x) has a discontinuity of the first kind at x = 0
(D) f(x) has a discontinuity of the second kind at x = 0

2. Which of the following is the value of the n^{th} derivative of log(1 + x) at x = 0 ?
(A) $(-1)^n n!$
(B) $(-1)^{n-1}(n-1)!$
(C) $(-1)^{n-1} n!$
(D) $(-1)^n (n-1)!$

3. The coefficient of x^5 in the Maclaurin's expansion of tan x is :
(A) 1
(B) $\frac{1}{3}$
(C) $\frac{1}{15}$
(D) $\frac{2}{15}$

4. $\lim_{x \to 1}\left(\frac{x}{x-1} - \frac{1}{\log x}\right) =$
(A) $\frac{1}{2}$
(B) $-\frac{1}{2}$
(C) $\frac{1}{3}$
(D) $-\frac{1}{3}$

5. Which of the following is true for the functions $u = x^3 - 3xy^2$, $v = 3x^2y - y^3$?
(A) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$
(B) $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
(C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$
(D) All of these

6. The radius of curvature of the circle $p = a(1 + \sin \psi)$ is :
(A) $a \sin \psi$
(B) $a \cos \psi$
(C) a
(D) None of these

7. Which of the following is/are an asymptote(s) of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$?
(A) $y = \frac{b}{a}x$
(B) $y = -\frac{b}{a}x$
(C) Both (A) and (B)
(D) Neither (A) nor (B)

8. The origin is a node on the curve $y^2 = ax^2 + bx^3$ if :
(A) $a > 0$
(B) $a = 0$
(C) $a < 0$
(D) None of these holds

9. The modulus and the amplitude of the complex number 'tan x – i' are respectively :

(A) $\cos x, \frac{\pi}{2} - x$

(B) $\sec x, \frac{\pi}{2} - x$

(C) $\sec x, x - \frac{\pi}{2}$

(D) $\cos x, x - \frac{\pi}{2}$

10. The product of all the values of $(1 + i)^{\frac{1}{3}}$ is :

(A) $\frac{1}{2}$

(B) $1 + i$

(C) i

(D) $1 - i$

11. If $i^{i^{i^{\cdots}}} = P + iQ$ and only principal values are considered, then $P^2 + Q^2 =$

(A) $e^{-P\pi}$

(B) $e^{-Q\pi}$

(C) $e^{-\pi}$

(D) e^{π}

12. If $\sin(P + iQ) = x + iy$, then $x^2 \operatorname{cosec}^2 P - y^2 \sec^2 p =$

(A) $\sinh P$

(B) $\cosh Q$

(C) 1

(D) 0

13. The condition that the line $y = mx + c$ may be a normal to the parabola $y^2 = 4ax$ is :

(A) $c = \frac{a}{m}$

(B) $am^2 + 2am + c = 0$

(C) $c = am^2$

(D) $a^2m^2 + 2am + c = 0$

14. The coordinates of the foci of the ellipse $3x^2 + 2y^2 = 6$ are :

(A) $\left(\pm\frac{1}{\sqrt{3}}, 0\right)$

(B) $\left(0, \pm\frac{1}{\sqrt{3}}\right)$

(C) $(\pm 1, 0)$

(D) $(0, \pm 1)$

15. The eccentricity of the hyperbola $2x^2 - 3y^2 = 15$ is :

(A) $\sqrt{\frac{5}{3}}$

(B) $\sqrt{\frac{5}{2}}$

(C) $\sqrt{\frac{3}{2}}$

(D) $\sqrt{\frac{15}{2}}$

16. The centre of the conic

$$5x^2 + 6xy + 5y^2 - 10x - 6y - 3 = 0$$

is :

(A) (0, 1)

(B) (1, 1)

(C) (1, 0)

(D) (0, 0)

17. The curve of intersection of two spheres is :

(A) a straight line

(B) a circle

(C) an ellipse

(D) a hyperbola

18. The locus of the tangent lines drawn from a given point to a sphere is :

(A) a sphere

(B) a cone

(C) a cylinder

(D) None of these

19. The equation of the cylinder whose generators are parallel to the z-axis and pass through the curve of intersection of $x^2+y^2+z^2=1$ and $x+y+z=1$ is :

(A) $x^2+y^2+xy-x-y=0$

(B) $x^2+y^2-xy+x+y=0$

(C) $x^2+y^2-xy+x-y=0$

(D) $x^2+y^2+xy+x+y=0$

20. The equation $z^2=4(1+x^2+y^2)$ represents :

(A) an ellipsoid

(B) a cone

(C) a hyperboloid of one sheet

(D) a hyperboloid of two sheets

21. The solution of the differential equation $(x+y+1)^2\frac{dy}{dx}=1$ is :

(A) $y=\tan^{-1}(x+y+1)+c$

(B) $x=\tan^{-1}(x+y+1)+c$

(C) $\tan(x+y+1)=y+c$

(D) $\cot(x+y+1)=y+c$

22. If P and Q are functions of x only or constants, then the general equation reducible to a linear differential equation is :

(A) $f(y)\frac{dy}{dx}+Pf'(y)=Q$

(B) $f'(y)\frac{dy}{dx}+Pf(y)=Q$

(C) $\frac{dy}{dx}+Pf(y)=Q$

(D) $f(y)\frac{dy}{dx}+Py=Q$

23. The particular integral of the differential equation $\frac{d^2y}{dx^2}+y=\text{cosec}\,x$ is :

(A) $\cos x\log\cos x-x\sin x$

(B) $\sin x\log\sin x-x\sin x$

(C) $\sin x\log\sin x-x\cos x$

(D) $\cos x\log\cos x-x\cos x$

24. Which of the following is not true for Bessel functions of different orders ?

(A) $J_0'(x)=J_1(x)$

(B) $\frac{d}{dx}\left[x^nJ_n(x)\right]=x^nJ_{n-1}(x)$

(C) $J_n'(x)+\frac{n}{x}J_n(x)=J_{n-1}(x)$

(D) $J_n'(x)-\frac{n}{x}J_n(x)=-J_{n+1}(x)$

25. The index of nilpotency of the matrix $\begin{bmatrix}0&2&4\\0&0&5\\0&0&0\end{bmatrix}$ is :

(A) 2

(B) 3

(C) 4

(D) 5

26. Which of the following is not true for any square matrices P, Q, R of the same order n ?

(A) $\text{Tr}(P+Q)=\text{Tr}(P)+\text{Tr}(Q)$

(B) $\text{Tr}(PQ)=\text{Tr}(QP)$

(C) $\text{Tr}(PQR)=\text{Tr}(QRP)$

(D) $\text{Tr}(PRQ)=\text{Tr}(PQR)$

27. For what value of 'a' is $\lambda^3 - 3\lambda^2 - 7\lambda + a = 0$ the characteristic equation of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$?

(A) $a = 5$

(B) $a = 3$

(C) $a = 11$

(D) $a = -11$

28. Which of the following is not true in general ?

(A) Every elementary matrix is non-singular

(B) Every non-singular matrix is a product of elementary matrices

(C) The inverse of an elementary matrix is an elementary matrix

(D) Rank(PQ) = max(Rank P, Rank Q)

29. Which of the following is not true ?

(A) A square matrix of order n is non-singular if and only if its rank is n

(B) The columns of a non-singular matrix are linearly dependent

(C) The columns of a matrix of order $m \times n$ are linearly dependent if and only if its rank is less than n

(D) A square matrix of order n has rank n if and only if its columns are linearly independent

30. For what values of 'b' the equations $x + y + z = 1$, $x + 2y + 4z = b$, $x + 2y + 4z = b^2$ have a solution ?

(A) $b = 1, 2$

(B) $b = 1, 3$

(C) $b = 2, 3$

(D) $b = 3, 4$

31. For what values of 'a' is the matrix $\begin{bmatrix} a \sin\theta & -a\cos\theta \\ a\cos\theta & a\sin\theta \end{bmatrix}$ orthogonal ?

(A) $a = 1$

(B) $a = -1$

(C) Both (A) and (B)

(D) Neither (A) nor (B)

32. The matrix $\begin{bmatrix} \alpha + i\beta & -\gamma + i\delta \\ \gamma + i\delta & \alpha - i\beta \end{bmatrix}$ is unitary if :

(A) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$

(B) $\alpha^2 + \beta^2 = \gamma^2 + \delta^2$

(C) $\alpha^2 + \gamma^2 = \beta^2 + \delta^2$

(D) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 0$

33. The equation whose roots are two times the roots of the equation $x^7 - 5x^4 + 13x^2 - 11 = 0$ is :

(A) $x^7 - 40x^4 + 416x^2 - 1408 = 0$

(B) $x^7 - 416x^4 + 40x^2 - 1408 = 0$

(C) $x^7 - 1408x^4 + 40x^2 - 416 = 0$

(D) $x^7 - 40x^4 - 416x^2 + 1408 = 0$

34. If α, β, γ are the roots of the equation $x^3 + 3x + 2 = 0$, then the equation whose roots are $(\beta - \gamma)^2$, $(\gamma - \alpha)^2$, $(\alpha - \beta)^2$ is :

(A) $x^3 + 18x^2 + 216x + 81 = 0$

(B) $x^3 - 18x^2 + 81x - 216 = 0$

(C) $x^3 + 18x^2 + 81x + 216 = 0$

(D) $x^3 - 18x^2 - 81x - 216 = 0$

35. The equation whose roots are the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ increased by 1 is :

(A) $x^3 - 9x^2 + 26x - 24 = 0$

(B) $x^3 + 9x^2 - 26x - 24 = 0$

(C) $x^3 - 9x^2 + 24x - 26 = 0$

(D) $x^3 + 9x^2 - 26x + 24 = 0$

36. If α, β, γ are the roots of the equation

$$x^3 - px^2 + qx - r = 0,$$

then $\alpha^{-1} + \beta^{-1} + \gamma^{-1} =$

(A) $\frac{p}{r}$

(B) $\frac{q}{r}$

(C) $\frac{r}{p}$

(D) $\frac{r}{q}$

37. The sum of the cubes of the roots of the equation $x^3 - 2x^2 + x - 1 = 0$ is :

(A) 2

(B) 5

(C) 8

(D) 10

38. The number of imaginary roots of the equation $x^9 - x^5 + x^4 + x^2 + 1 = 0$ is at least :

(A) 3

(B) 5

(C) 6

(D) 7

39. If α, β, γ are the roots of the equation

$$ax^3 + bx^2 + cx + d = 0,$$

then $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha) =$

(A) $\frac{ad - bc}{a^2}$

(B) $\frac{c^2 - 2dc}{a^2}$

(C) $\frac{b^2 - 2ac}{d^2}$

(D) None of these

40. The number of rational roots of the equation $3x^3 - x^2 + 3x - 1 = 0$ is :

(A) 1

(B) 2

(C) 3

(D) None of these

41. Which of the following is not true for any two real numbers x and y ?

(A) $\|x| + |y\| \le |x + y|$

(B) $|x+y| = |x| + |y|$ iff $xy \ge 0$

(C) $|x+y| < |x| + |y|$ iff $xy < 0$

(D) $\|x| - |y\| \le |x - y|$

42. The sequence $\{b_n\}$ defined by

$$b_n = \frac{n}{(n!)^{\frac{1}{n}}}, \; n = 1,2,3,.....$$

converges to :

(A) 0

(B) 1

(C) e

(D) $\frac{1}{e}$

43. Which of the following series is convergent ?

(A) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(B) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$

(C) $\sum_{n=1}^{\infty} \sin \frac{1}{n}$

(D) $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$

44. Which of the following is true for the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$?

(A) The series is absolutely convergent

(B) The series is divergent

(C) The series is conditionally convergent

(D) None of these

45. Which of the following is true for the function

$f(x) = 0$, when x is rational

$= 1$, when x is irrational

in any interval [a, b] ?

(A) $\int_a^b f(x)dx = 0$

(B) $\int_a^b f(x)dx = b - a$

(C) f(x) is integrable on [a, b]

(D) None of these

46. For any two partitions P_1 and P_2 of [a, b] and for any bounded function f(x) defined on [a, b], which of the following is not true ?

(A) $L(P_1, f) \leq U(P_2, f)$

(B) $L(P_1 \cup P_2, f) \geq L(P_1, f)$

(C) $U(P_1 \cup P_2, f) \leq U(P_2, f)$

(D) $U(P_1, f) \leq U(P_1 \cup P_2, f)$

47. The value of $\int_0^3 \{x + [x]\}dx$, where [x] is the greatest integer function, is :

(A) $\frac{5}{2}$

(B) $\frac{1}{2}$

(C) $\frac{15}{2}$

(D) None of these

48. Which of the following is true for the function

$f(x) = 2x \sin \frac{1}{x} - \cos \frac{1}{x}$, $x \in (0, 1]$

$= 0$, $x = 0$?

(A) f(x) is continuous on [0, 1]

(B) f(x) is differentiable on [0, 1]

(C) f(x) is integrable on [0, 1]

(D) None of these

49. The value of $\int_0^{\pi} \int_0^{a \sin \theta} r dr d\theta$ is :

(A) $\frac{\pi a^2}{4}$

(B) $\frac{a^2}{6}$

(C) $\frac{5\pi a^3}{8}$

(D) $\frac{\pi a^2}{6}$

50. Which of the following is not true ?

(A) For any constant vector $\underset{a}{\longrightarrow}$, $\nabla \cdot \underset{a}{\longrightarrow} = 0$

(B) For any vector function

$$\underset{v}{\longrightarrow}, \nabla \times (\nabla \times \underset{v}{\longrightarrow}) = \nabla(\nabla \cdot \underset{v}{\longrightarrow}) + \nabla^2 \underset{v}{\longrightarrow}$$

(C) For any scalar function ϕ, $\nabla \times \nabla\phi = \underset{0}{\longrightarrow}$

(D) For any vector function

$$\underset{v}{\longrightarrow}, \nabla \cdot (\nabla \times \underset{v}{\longrightarrow}) = 0$$

51. For what value of 'a' is the function

$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}, (x, y) \neq (0, 0)$$
$$= a, (x, y) = (0, 0)$$

continuous at (0, 0) ?

(A) 0

(B) 1

(C) –1

(D) None of these

52. The saddle points of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$

are :

(A) (1, 2), (–1, 2)

(B) (1, 2), (1, –2)

(C) (1, –2), (–1, –2)

(D) (–1, 2), (1, –2)

53. Which of the following is true for any normal subgroup H of a group G ?

(A) $xH = Hx, \forall x \in G$

(B) $x^{-1}hx \in H, \forall x \in G, h \in H$

(C) $(xH)(yH) = (xy)H, \forall x, y \in G$

(D) All of the above

54. If K is the kernel of a homomorphism from a group G onto the group G', then which of the following is true in general ?

(A) $\frac{G}{K} \cong G'$

(B) $\frac{G'}{K} \cong G$

(C) $K = \{e\}$

(D) $K = G$

55. Which of the following is/are true for any ring R and for any ideal I of R ?

(A) $f : R \to \frac{R}{I}$ defined by $f(x) = x + I, \forall x \in R$ is a homomorphism with kernel I

(B) I = (0) or R if R is a field

(C) If R is an integral domain, then so is $\frac{R}{I}$

(D) All the above

56. For any two ideals U and V of a ring R, which of the following is/are not an ideal(s) of R ?

(A) $U \cup V$

(B) $U \cap V$

(C) $U + V$

(D) All the above

57. Which of the following is not a subspace of the real linear space of all the bounded continuous real valued functions on $[-1, 1]$?

(A) The set of all f such that $f(0) = 0$

(B) The set of all f such that

$$f(x) \geq 0, \forall x \in [-1, 1]$$

(C) The set of all f such that f is differentiable

(D) The set of all f such that $\int_{-1}^{1} f(x)dx = 0$

58. Which of the following sets in the linear space R^3 over R is linearly independent ?

(A) $\{(2, 1, 2), (8, 4, 8)\}$

(B) $\{(1, 2, 0), (0, 3, 1), (-1, 0, 1)\}$

(C) $\{(-1, 2, 1), (3, 0, -1), (-5, 4, 3)\}$

(D) None of these

59. If P is the real linear space of all polynomials f(x) with real coefficients defined on [0, 1] and D and T are two transformations on P defined as $D[f(x)] = \frac{d}{dx} f(x), \forall f(x) \in P$ and $T[f(x)] = xf(x), \forall f(x) \in P$, then which of the following is true ?

(A) $DT = TD$

(B) $DT \neq TD$

(C) $DT - TD \neq I$, the identity transformation

(D) None of these

60. Which of the following is/are true for any two subspaces L and M of a finite dimensional linear space X over the field F with dual space X^* ?

(A) $\dim(L \cup M) = \dim(L) + \dim(M) - \dim(L \cap M)$

(B) $\dim\left(\frac{X}{L}\right) = \dim(X) - \dim(L)$

(C) $\dim(X) = \dim(X^*)$

(D) All the above

Sr. No. 099

ENTRANCE TEST-2017

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES
STATISTICS
Option—(i) Statistics & Option—(ii) Mathematics

Question Booklet Series **A**

Total Questions : 60 + 60
Time Allowed : 70 Minutes

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be 'Negative Marking' for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

Option—(i) : Statistics

1. The most appropriate average to determine the size of oranges on a tree is:
   (A) Arithmetic Mean
   (B) Median
   (C) Mode
   (D) Geometric mean

2. The variance of first n natural numbers is:
   (A) $\frac{n^2-1}{12}$
   (B) $\frac{n+1}{2}$
   (C) $\frac{n(n+1)}{2}$
   (D) $\frac{n(n+1)(2n+1)}{6}$

3. In a positively skewed distribution:
   (A) Mean > Mode > Median
   (B) Mean > Median >Mode
   (C) Mean < Median < Mode
   (D) Mean < Mode < Median

4. The first quartile divides a frequency distribution in the ratio:
   (A) 4 : 1
   (B) 1 : 4
   (C) 3 : 1
   (D) 1 : 3

5. The coefficient of correlation is independent of change of:
   (A) Origin only
   (B) Scale only
   (C) Origin and scale
   (D) None

6. If cov (x,y) = $\sigma_x\sigma_y$, then:
   (A) r = 1
   (B) r = 0
   (C) r = 2
   (D) r = –1

7. If r = 0, the lines of regression are:
   (A) Coincide
   (B) Perpendicular to each other
   (C) Parallel to each other
   (D) Do not exist

8. The regression coefficients are b_2 and b_1. Then th correlation Coefficient r is:
   (A) $\frac{b_1}{b_2}$
   (B) $\frac{b_2}{b_1}$
   (C) $b_1 b_2$
   (D) $\pm\sqrt{b_1 b_2}$

9. What is the chance that non-leap year should have fifty-three Sundays?
   (A) $\frac{1}{7}$
   (B) $\frac{2}{7}$
   (C) $\frac{6}{7}$
   (D) $\frac{5}{7}$

10. A problem in Statistics is given to three students X, Y, Z whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. The probability that problem will be solved is:
   (A) $\frac{1}{3}$
   (B) $\frac{3}{4}$
   (C) $\frac{1}{24}$
   (D) $\frac{1}{4}$

SEAL

11. If two dice are thrown, what is the probability that the sum is neither 7 nor 11?

(A) $\frac{7}{9}$

(B) $\frac{2}{9}$

(C) $\frac{1}{108}$

(D) $\frac{107}{108}$

12. The axiomatic approach to probability was proposed by:

(A) Von-Misses

(B) Laplace

(C) Thomas Bayes

(D) A.N. Kolmogorov

13. Suppose that a game is to be played with a single die assumed fair. In this game a player wins $ 20 if a 2 turns up, $ 40 if a 4 turns up; loses $30 if a 6 turns up; while the player neither wins nor loses if any other face turns up. The expected sum of money to be won is:

(A) $ 30

(B) $ 12

(C) $ 5

(D) $ 42

14. The moment generating function of geometric distribution is given by:

(A) $\frac{q}{(1-pe^t)}$

(B) $\frac{q}{(1+pe^t)}$

(C) $\frac{p}{(1+qe^t)}$

(D) $\frac{p}{(1-qe^t)}$

15. The variance of a binomial distribution $\binom{10}{x}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{10-x}$; $x = 0, 1, \ldots 10$ is:

(A) 1.6

(B) 2

(C) 5

(D) 8

16. A family of distributions which has mean always less than its variance is:

(A) Binomial distribution

(B) Negative binomial distribution

(C) Normal distribution

(D) Hyper-geometric distribution

17. Let $X \sim N(0, 1)$ and $Y \sim N(0, 1)$ be independent random variables. Then the distribution of X/Y is:

(A) Gamma distribution

(B) Standard Normal distribution

(C) Beta distribution

(D) Standard Cauchy distribution

18. If the moment generating function of a distribution is given by $\left(1-\frac{t}{\lambda}\right)^{-r}$, then mean of the distribution is:

(A) $\frac{\lambda}{r}$

(B) $\frac{r}{\lambda^2}$

(C) $\frac{r}{\lambda}$

(D) $\frac{\lambda^2}{r}$

19. Let $X \sim \beta_1(a, b)$, then variance of the distribution is:

(A) $\frac{ab}{(a+b)^2(a+b+1)}$

(B) $\frac{ab}{(a-b)^2(a+b+1)}$

(C) $\frac{ab}{(a+b)^2(a+b-1)}$

(D) $\frac{ab}{(a-b)^2(a+b-1)}$

20. The probability that a normal variate will lie within the range $\mu \pm \sigma$ is:
(A) 0.9973
(B) 0.0027
(C) 0.9544
(D) 0.6826

21. If the sample mean is closer to the population mean, then:
(A) Sampling error is larger
(B) Sampling error is smaller
(C) Sampling error equals to unity
(D) None of these

22. A random sample of 500 pineapples was taken from a large consignment and 65 were found to be bad. The standard error of the proportion of bad ones in a sample of this size is:
(A) 0.013
(B) 0.015
(C) 0.130
(D) 0.150

23. A randomly selected sample of 1,000 college students was asked whether they had ever used the drug Ecstasy. Sixteen percent (16%) of the 1,000 students surveyed said they had. Which one of the following statements about the number 0.16 is correct?
(A) It is a margin of error
(B) It is a population proportion
(C) It is a sample proportion
(D) It is a randomly chosen number

24. In hypothesis testing, a Type -II error occurs when:
(A) The null hypothesis is not rejected when the null hypothesis is true.
(B) The null hypothesis is rejected when the null hypothesis is true.
(C) The null hypothesis is not rejected when the alternative hypothesis is true.
(D) The null hypothesis is rejected when the alternative hypothesis is true.

25. Degrees of freedom for Chi-square in case of contingency table of order 3×4 is:
(A) 12
(B) 9
(C) 8
(D) 6

26. The mode of Chi-square distribution with n d.f is:
(A) n–2
(B) n–1
(C) 2n
(D) n

27. F-distribution is used for testing:
(A) Goodness of fit
(B) Equality of several means
(C) Independence of attributes
(D) Equality of proportions

28. t-distribution is used to test:
(A) The significance of the difference between two sample means
(B) Equality of population variances
(C) The independence of attributes
(D) Equality of proportions

29. The difference between a sample estimate and the population parameter from a complete count is termed as:
(A) Human error
(B) Sampling error
(C) Non-sampling error
(D) None of the above

30. A selection procedure of a sample having no involvement of probability is known as:
(A) Purposive sampling
(B) Judgement sampling
(C) Subjective sampling
(D) All the above

31. A complete list of sampling units which represents the population to be covered is known as:

(A) Sample size

(B) Sampling unit

(C) Sampling frame

(D) Sample Space

32. The variance of 11, 11, 11, 11,11 is:

(A) 0

(B) 1

(C) 11

(D) $\sqrt{11}$

33. Under cost optimum allocation, the allocation of sample size to different strata depends on:

(A) Stratum size

(B) Sampling cost per unit in the stratum

(C) Stratum variability

(D) All of these

34. A sampling technique in which only the first unit is selected with the help of random numbers and the rest get selected automatically according to some pre-designed pattern is known as:

(A) Simple random sampling

(B) Stratified sampling

(C) Systematic sampling

(D) Cluster sampling

35. Variance of $\bar{y}_{st}$ under random sampling, proportional allocation and optimum allocation hold the correct inequality as:

(A) $V(\bar{y}_{St})_{opt} \leq V(\bar{y}_{St})_{prop}\, V(\bar{y}_{St})_{SRS}$

(B) $V(\bar{y}_{St})_{SRS} \leq V(\bar{y}_{St})_{prop}\, V(\bar{y}_{st})_{opt}$

(C) $V(\bar{y}_{St})_{prop} \leq V(\bar{y}_{St})_{opt}\, V(\bar{y}_{St})_{SRS}$

(D) None of these

36. The sampling variance of systematic sample mean $\bar{y}_{Sy}$ is given by:

(A) $V(\bar{y}_{Sy}) = \frac{(N-1)S^2}{N} - \frac{k(n-1)}{N} S^2_{Wsy}$

(B) $V(\bar{y}_{Sy}) = \frac{(N-1)S^2}{N} + \frac{k(n-1)}{N} S^2_{Wsy}$

(C) $V(\bar{y}_{Sy}) = \frac{(N-1)S^2}{N} - \frac{k(n+1)}{N} S^2_{Wsy}$

(D) None of these

37. Randomization in an experiment helps to reduce:

(A) Systematic influences

(B) Human biases

(C) Dependence among observations

(D) All of these

38. The different procedures whose effects are to be measured and compared are known as:

(A) Experiment

(B) Treatment

(C) Experimental unit

(D) Yield

39. In an analysis of variance problem involving 3 treatments and 10 observations per treatment, SSE = 399.6. The MSE for this situation is

(A) 133.2

(B) 13.32

(C) 14.8

(D) 30.0

40. A researcher reports an F-ratio with df equal to (3, 36) for an independent measure of experiment. How many treatment conditions were compared in this experiment?

(A) 3

(B) 4

(C) 36

(D) 39

31. A complete list of sampling units which represents the population to be covered is known as:

(A) Sample size

(B) Sampling unit

(C) Sampling frame

(D) Sample Space

32. The variance of 11, 11, 11, 11,11 is:

(A) 0

(B) 1

(C) 11

(D) $\sqrt{11}$

33. Under cost optimum allocation, the allocation of sample size to different strata depends on:

(A) Stratum size

(B) Sampling cost per unit in the stratum

(C) Stratum variability

(D) All of these

34. A sampling technique in which only the first unit is selected with the help of random numbers and the rest get selected automatically according to some pre-designed pattern is known as:

(A) Simple random sampling

(B) Stratified sampling

(C) Systematic sampling

(D) Cluster sampling

35. Variance of $\bar{y}_{st}$ under random sampling, proportional allocation and optimum allocation hold the correct inequality as:

(A) $V(\bar{y}_{St})_{opt} \leq V(\bar{y}_{St})_{prop}\ V(\bar{y}_{St})_{SRS}$

(B) $V(\bar{y}_{St})_{SRS} \leq V(\bar{y}_{St})_{prop}\ V(\bar{y}_{st})_{opt}$

(C) $V(\bar{y}_{St})_{prop} \leq V(\bar{y}_{St})_{opt}\ V(\bar{y}_{St})_{SRS}$

(D) None of these

36. The sampling variance of systematic sample mean $\bar{y}_{Sy}$ is given by:

(A) $V(\bar{y}_{Sy}) = \frac{(N-1)S^2}{N} - \frac{k(n-1)}{N} S^2_{Wsy}$

(B) $V(\bar{y}_{Sy}) = \frac{(N-1)S^2}{N} + \frac{k(n-1)}{N} S^2_{Wsy}$

(C) $V(\bar{y}_{Sy}) = \frac{(N-1)S^2}{N} - \frac{k(n+1)}{N} S^2_{Wsy}$

(D) None of these

37. Randomization in an experiment helps to reduce:

(A) Systematic influences

(B) Human biases

(C) Dependence among observations

(D) All of these

38. The different procedures whose effects are to be measured and compared are known as:

(A) Experiment

(B) Treatment

(C) Experimental unit

(D) Yield

39. In an analysis of variance problem involving 3 treatments and 10 observations per treatment, SSE = 399.6. The MSE for this situation is

(A) 133.2

(B) 13.32

(C) 14.8

(D) 30.0

40. A researcher reports an F-ratio with df equal to (3, 36) for an independent measure of experiment. How many treatment conditions were compared in this experiment?

(A) 3

(B) 4

(C) 36

(D) 39

41. A basic feasible solution of a LPP is said to be _____ if at least one of the basic variables is zero:

(A) Degenerate

(B) Non-degenerate

(C) Infeasible

(D) Unbounded

42. Any feasible solution which optimizes the objective function of the LPP is called its:

(A) Solution

(B) Non-basic variables

(C) Optimal solution

(D) Basic feasible solution

43. While solving the LPP graphically, the area bounded by the constraints is called:

(A) Feasible region

(B) Infeasible region

(C) Unbounded solution

(D) None of these

44. The solution to a transportation problem with m-rows (supplies) and n-columns (destinations) is feasible if the number of possible allocations are:

(A) m+n

(B) m x n

(C) m+n+1

(D) m+n-1

45. The sales of a Departmental store on Eid and Christmas are associated with the components of a time series:

(A) Secular trend

(B) Seasonal variation

(C) Cyclical variation

(D) Irregular variation

46. Given that annual trend equation $Y_c = 40 + 2X$ with origin as 1985; X-units = 6 months and Y-unit = annual production of computers. The monthly trend equation is:

(A) $Y_c = 3.33 + 0.03\ X$

(B) $Y_c = 3.33 + 0.01\ X$

(C) $Y_c = 3.39 + 0.02\ X$

(D) $Y_c = 3.33 + 0.21\ X$

47. Which of the following satisfies the time reversal test?

(A) Walsh price index

(B) Paasche's index

(C) Laspeyre's index

(D) Fisher's ideal formula

48. The arithmetic mean of Laspeyre's index and Paasche's index is:

(A) Fisher's ideal method

(B) Kelley's method

(C) Bowley Dorfish Method

(D) Marshall Egdeworth method

49. The child bearing age in India is:

(A) 15-29 years

(B) 15-39 years

(C) 15-34 years

(D) 15-49 years

50. The registration of births, deaths and marriages are:

(A) Fancy of society

(B) Legal documents

(C) Part of medical research

(D) All of these

51. The sum of age specific fertility rate is known as:

(A) Total fertility rate

(B) General fertility rate

(C) Crude birth rate

(D) Net reproduction rate

52. For mathematical manipulations, the radix is:

(A) 1000

(B) 100

(C) 1

(D) Either (A) or (B)

53. Assignable causes:

(A) are not as important as natural causes

(B) are causes of variation that can be identified and removed

(C) are also referred to as chance causes

(D) are within the limits of a control chart

54. Central tendency of a process is monitored in:

(A) R- chart

(B) C- chart

(C) S- chart

(D) X- chart

55. The term LCD stands for:

(A) Liquefy Crystal Display

(B) Liquid Crystal Display

(C) Liquid Card Display

(D) Liquid Clear Display

56. A chart that controls the total number of defects divided by the sample size is known as:

(A) R chart

(B) np chart

(C) C chart

(D) p chart

57. A property of a point estimator that occurs whenever larger sample sizes tend to provide point estimates closer to the population parameter is known as:

(A) Efficiency

(B) Unbiased sampling

(C) Consistency

(D) Relative estimation

58. The purpose of statistical inference is to provide information about the:

(A) Sample based upon information contained in the population

(B) Population based upon information contained in the sample

(C) Population based upon information contained in the population

(D) Mean of the sample based upon the mean of the population

59. Let $x_1, x_2, \ldots, x_n$ be a random sample from $N(\mu, 1)$, then the unbiased estimator of $\frac{1}{n}\sum_{i=1}^{n} x_i^2$ is

(A) $\mu - 1$

(B) $\mu^2 - 1$

(C) $\mu + 1$

(D) $\mu^2 + 1$

60. Let $X \sim U(0, \theta)$, then sufficient estimator for θ is:

(A) $\sum_{i=1}^{n} X_i$

(B) $\prod_{i=1}^{n} X_i$

(C) $\max(X_1, X_2, \ldots\ldots, X_n)$

(D) $\min(X_1, X_2, \ldots, X_n)$

Option—(ii) : Mathematics

1. The value of the limit $\lim_{x\to 0}\left[1+\frac{x}{a}\right]^{a/x}$ is :

(A) $e^{a/x}$

(B) e^{ax}

(C) $\log a$

(D) e

2. For what value of k, $\lim_{x\to 0}\frac{|x|}{x^2+2x}=k$?

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) zero

(D) k does not exist

3. The points of discontinuity of the function $f(x)=\frac{\log x}{x-1}$ are :

(A) $x = 1, 0$

(B) $x \geq 0, x = 1$

(C) $x \leq 0, x = 1$

(D) Everywhere continuous

4. For $f(x) e^x$ and $g(x) = e^{-x}$, the value of c in the interval [a, b] using Cauchy's Mean value theorem is :

(A) ab

(B) $\frac{ab}{2}$

(C) $a + b$

(D) $\frac{a+b}{2}$

5. For the relation, $y = x - x^2$, the rate of change of y^2 with respect to x^2 is :

(A) $1 - 3x + 2x^2$

(B) $1 + 3x - 2x^2$

(C) $1 + 2x - 3x^2$

(D) $1 - 2x + 3x^2$

6. The expansion of e^x in the ascending powers of $(x-1)$ is :

(A) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....$

(B) $e\left[1+(x-1)+\frac{(x-1)^2}{2!}+\frac{(x-1)^3}{3!}+....\right]$

(C) $e\left[1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....\right]$

(D) $1+(x-1)+\frac{(x-1)^2}{2!}+\frac{(x-1)^3}{3!}+....$

7. For $u = x^3 + y^3 + z^3 + 3xyz$, the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$ is :

(A) u

(B) 3u

(C) 6u

(D) zero

8. The envelope of the ellipse $x = a\sin(\theta-\alpha)$, $y = b\cos\theta$ is ..... ; where α is a parameter.

(A) $x^2 = a^2$

(B) $x^2 = 0$

(C) $x = a$

(D) does not exist

9. The partial derivative of $f(x, y, z)=e^{1-x\cos y}+ze^{\frac{1}{(1+y^2)}}$ with respect to x at $(1, 0, \pi)$ is :

(A) -1

(B) $-\frac{1}{e}$

(C) zero

(D) π

10. The angle between the radius vector and the tangent of the curve $r = a(1 + \sin\theta)$ at $\theta = \pi/6$

(A) $\pi/2$

(B) $\pi/3$

(C) $\pi/4$

(D) $\pi/6$

11. The equation of the tangent to the curve $y = x^3$ at the point (2, 8) is :
(A) $Y = 12X + 16$
(B) $Y = 16X - 12$
(C) $Y = 12X - 16$
(D) $Y = 16X + 12$

12. The function $\frac{\log x}{x} (x > 0)$ has :
(A) maximum value = e
(B) minimum value = e
(C) maximum value = 1/e
(D) minimum value = 1/e

13. The radius of curvature at any point of the tractrix $s = c \log\sec \psi$ is :
(A) $c \tan \psi$
(B) $c \sec \psi$
(C) $c \cos \psi$
(D) $c \cot \psi$

14. If m_1 and m_2 represent the slopes of tangent and normal to the ellipse $4x^2 + 3y^2 = 24$ at the point $(\sqrt{3}, 2)$, then :
(A) $m_1 = -\frac{\sqrt{3}}{2} \& m_2 = \frac{2}{\sqrt{3}}$
(B) $m_1 = -\frac{2}{\sqrt{3}} \& m_2 = \frac{\sqrt{3}}{2}$
(C) $m_1 = -\frac{2}{\sqrt{3}} \& m_2 = \frac{2}{\sqrt{3}}$
(D) $m_1 = \frac{2}{\sqrt{3}} \& m_2 = \frac{\sqrt{3}}{2}$

15. The difference of the focal distances of any point on the hyperbola is
I. constant
II. equal to the length of transverse axis
III. zero
(A) only I is correct
(B) only II is correct
(C) I and II are correct
(D) only III is correct

16. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$ is :
(A) 30°
(B) 45°
(C) 60°
(D) 90°

17. The value of $\frac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5}$ is
(A) zero
(B) –1
(C) 1
(D) i

18. The value of log(–5) is equal to :
(A) $-\log 5 + \pi i$
(B) $-\log 5 + 2\pi i$
(C) $\log 5 + 2\pi i$
(D) $\log 5 + \pi i$

19. For what value of k, $\log(-i) = \frac{k\pi}{2}$?
(A) 1
(B) –1
(C) i
(D) –i

20. The pair of equations of the tangent to the parabol $y^2 = 4ax$ at the ends of its latus rectum are:
(A) $y = \pm (x + a)$
(B) $y = x \pm a$
(C) $y = \pm 2(x + a)$
(D) $y = 2(x \pm a)$

21. Let (α, β) represent the centre and radius of th sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y - 6z = 1$, then
(A) $\alpha = \left(1, -\frac{1}{2}, 3\right), \beta = 2$
(B) $\alpha = \left(\frac{1}{2}, -1, \frac{3}{2}\right), \beta = 2$
(C) $\alpha = \left(\frac{1}{2}, -1, \frac{3}{2}\right), \beta = 4$
(D) $\alpha = \left(1, -\frac{1}{2}, 3\right), \beta = 4$

22. The necessary and sufficient conditon that the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ may have three mutually perpendicular generators is :

(A) $f + g + h = 0$

(B) $a + b + c = f + g + h$

(C) $a + b + c = 0$

(D) $a + b + c = 1$

23. The plane $lx + my + nz = p$ touches the conicoid $ax^2 + by^2 + cz^2 = 1$ if :

(A) $\frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c} = p^2$

(B) $l^2 + m^2 + n^2 = p^2$

(C) $\frac{a}{l^2} + \frac{b}{m^2} + \frac{c}{n^2} = p^2$

(D) $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = p$

24. The polar plane of the point $(2, -3, 4)$ with respect to the conicoid $x^2 + 2y^2 + z^2 = 4$ is :

(A) $2x - 6y + 4z = -17$

(B) $x + 2y + z = 17$

(C) $x - 3y + 2z = 2$

(D) $x + 2y + z = 0$

25. For the function

$$f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0; & (x, y) = (0, 0) \end{cases}$$

the value of f_{yx} at $(0, 0)$ is :

(A) -1

(B) zero

(C) 1

(D) f_{yx} do not exist

26. Let $f(x, y) = \begin{cases} xy \tan\frac{y}{x}; & (x, y) \neq (0, 0) \\ 0; & (x, y) = (0, 0) \end{cases}$; then

(A) $xf_x + yf_y + xy = 2f$

(B) $xf_y + yf_x = 2f$

(C) $xf_x + yf_y = 2f$

(D) $xf_x - yf_y = 2f$

27. The solution of the differential equation $y'' + y = 0$ satisfying $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = 2$ is

(A) $\cos x + 2 \sin x$

(B) $\cos x + \sin x$

(C) $2\cos x + \sin x$

(D) $2(\cos x + \sin x)$

28. If $\sin x$ is the integrating factor of $y' + Py = Q$, then P is equal to :

(A) $\sin x$

(B) $\log \sin x$

(C) $\log \cos x$

(D) $\cot x$

29. The solution of $\frac{d^2y}{dx^2} = 0$ represents a :

(A) point

(B) straight line

(C) parabola

(D) circle

30. The number of generators in an infinite cyclic group of Integers is :

(A) 1

(B) 2

(C) finitely many

(D) Infinity

31. In a group G, if every element is its own inverse, then G is :

(A) cyclic

(B) finite

(C) abelian

(D) non-abelian

32. Let a and b are two elements of a group G such that order of a is 5 and $a^3b = ba^3$, then

(A) $ab = ba$

(B) $a = b^{-1}$

(C) $O(b) = 3$

(D) None of these

33. If C(G) and N(a) respectively denote centre of a group G and Normalizer of an element a in G, then :

(A) $C(G) = N(a)$

(B) $N(a) \subseteq C(G)$

(C) $C(G) \subseteq N(a)$

(D) $C(G) \neq N(a)$

34. If a = (1 3 5) (1 2) and b = (1 5 7 9) are two permutations, then $a^{-1}ba$ is equal to :

(A) (1 7 5 9)

(B) (3 7 9 5)

(C) (3 7 9 1)

(D) None of these

35. In Z_{18}, the subgroups $< 3 >$ and $< 15 >$ are associated as

(A) $< 3 > \subseteq < 15 >$

(B) $< 15 > \subseteq < 3 >$

(C) $< 15 > = < 3 >$

(D) None of these

36. Which of the following statements is incorrect?

(A) Only ideals of a field F are (0) and F itself

(B) Every field is a division ring

(C) A finite integral domain is a field

(D) None of these

37. Find values of b such that the vectors $< -11, b, 2 >$ and $< b, b^2, b >$ are orthogonal :

(A) 0, 3, –3

(B) 0, 11, 3

(C) 0, –11, 2

(D) 0, 2, –2

38. Which of the following statements is incorrect?

(A) A monotonic sequence is never oscillatory.

(B) If a sequence $\{a_n\}$ converges to l, then $\{|a_n|\}$ converges to $|l|$

(C) A sequence cannot converge to more than one limit.

(D) The sequence with nth term $a_n = \left(1+\frac{1}{n}\right)^n$ converges to 1

39. The geometric series $1 + x + x^2 + \ldots$ to ∞ is

I. Convergent if $|x| < 1$

II. Divergent if $x \geq 1$

III. Oscillates infinitely if $x < -1$

(A) Only I is correct

(B) I and II are correct

(C) I and III are correct

(D) I, II and III are correct

40. The series $\sum_{n=1}^{\infty} n^{-p}$ is :

(A) Convergent for $0 < p \leq 1$

(B) Divergent for $0 < p \leq 1$

(C) Convergent for all p

(D) Divergent for all p

41. The Dirichlet's function defined by

$$f(x) = \begin{cases} 1, & x \text{ rational} \\ -1 & x \text{ irrational} \end{cases}$$ is

(A) continuous for every real x

(B) discontinuous for every real x

(C) continuous for rational numbers

(D) continuous for irrational numbers

42. Identify the incorrect statement.

(A) Every bounded function is Riemann Integrable.

(B) A continuous function $f : [a, b] \to R$ is Riemann Integrable in [a, b].

(C) A monotone function $f : [a, b] \to R$ is Riemann Integrable in [a, b].

(D) If the set of points of discontinuity of a bounded function $f : [a, b] \to R$ is finite, then it is R-integrable.

43. For $f(x) = x$, $x \in [0, 1]$ and $P = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ be a partition of [0, 1], then $U(P, f, x) + L(P, f, x) =$

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) 1

(D) zero

44. For what value of k, $\int_0^4 [x]\,dx = k$?

(A) $k = 4$

(B) $k = 6$

(C) $k = 8$

(D) None of these

45. If $f : [a, b] \to R$ is a bounded function, then for $\varepsilon > 0$ and $\delta > 0$ such that

I. $U(P, f) < \int_a^{\bar{b}} f(x)dx + \varepsilon$

II. $L(P, f) > \int_{\underline{a}}^{b} f(x)dx - \varepsilon$ for each $P \in [a, b]$, $\|x\| < \delta$

(A) Only I is correct

(B) Only II is correct

(C) Both I and II are correct

(D) Both I and II are incorrect

46. Which of the following is not correct?

(A) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

(B) $\int_0^\infty e^{-x^2}\,dx = \frac{1}{2}\sqrt{\pi}$

(C) $\Gamma(n) = n\Gamma(n-1)$

(D) $B(m, n) = B(n, m)$

47. The value of the integral $\int x^3 e^{-x}\,dx$ is

(A) $-e^{-x}(x^3 + 3x^2 + 6x + 6) + c$

(B) $e^{-x}(x^3 + 3x^2 + 6x + 6) + c$

(C) $-e^{-x}(x^3 - 3x^2 + 6x - 6) + c$

(D) None of these

48. The value of the integral $\int_{y=4}^{5} \int_{x=0}^{1} 4xy\,dx\,dy$ is

(A) zero

(B) 1

(C) 9

(D) None of these

49 If $\int f(x)\cos x\,dx = \frac{1}{2}\{f(x)\}^2 + c$, then $f(x)$ is :

(A) $x + c$

(B) $\sin x + c$

(C) $\cos x + c$

(D) c

50. For what value of k, $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx = k$?

(A) $\pi/2$

(B) π

(C) $\pi/4$

(D) $\pi^2/2$

51. Given that the roots of the $4x^3 + 12x^2 + 11x + 3 = 0$ are in A.P., its roots are :

(A) $\left(-1, -\frac{1}{2}, -\frac{3}{2}\right)$

(B) $\left(1, \frac{1}{2}, \frac{3}{2}\right)$

(C) $\left(-1, -\frac{3}{2}, -\frac{5}{2}\right)$

(D) None of these

52. If α, β are the two distinct roots of

$$x^4 - 14x^3 + 73x^2 - 168x + 144 = 0$$

such that they occur in pairs, then $\alpha - \beta =$ ..... is:

(A) zero

(B) 1

(C) 7

(D) None of these

53. On multiplying the roots of the equation $x^4 + 3x^3 - x^2 + x - 1 = 0$ by 2, the resulting transformed equation is

(A) $y^4 + 6y^3 - 2y^2 + 2y - 2 = 0$

(B) $y^4 + 6y^3 - 4y^2 + 8y - 16 = 0$

(C) $y^4 - 6y^3 + 4y^2 - 8y + 16 = 0$

(D) None of these

54. If α, β and γ are the roots of the equation

$x^3 - ax^2 + bx - c = 0$, then $\sum \alpha^3$ is

(A) $a^3 - 3ab + 3c$

(B) $a^3 + 3ab - 3c$

(C) $a^3 - 2ab + c$

(D) $a^3 - ab + c$

55. The rank of the matrix $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{pmatrix}$ is :

(A) 1

(B) 2

(C) 3

(D) 4

56. For what values of k and l, $A = B^{-1}$? where,

$$A = \begin{pmatrix} k & 0 & -1 \\ 0 & 1 & l \\ 0 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(A) $k = 1$ & $l = 1$

(B) $k = 0$ & $l = 1$

(C) $k = 1$ & $l = 0$

(D) None of these

57. The following system of equations has

$2x + 6y = -11$

$6x + 20y - 6z = -3$

$6y - 18z = -1$

(A) unique solution

(B) no solution

(C) infinite number of solutions

(D) finite number of solutions

58. Which of the following statements is incorrect?

(A) If A is symmetric, then so is AA' and $A'A$.

(B) Every sq. matrix can be uniquely expressed as the sum of symmetric and skew symmetric matrices.

(C) If A is a Hermition matrix, then so is AA^H and A^HA.

(D) If A and B are symmetric, then so is AB and BA.

59. Find a matrix A such that $\left(2A^T + \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\right)^T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

and give its first row

(A) (2, –1)

(B) (0, 0)

(C) (–1/2, 1/2)

(D) (0, 1/2)

60. If $A\lambda^3 + B\lambda^2 + C\lambda + D = 0$ is the characteristic equation of the matrix $M = \begin{pmatrix} -1 & 0 & 2 \\ 0 & -2 & 6 \\ 0 & 0 & 3 \end{pmatrix}$. Then,

(A) $A = 1, B = 0, C = -7, D = 6$

(B) $A = 1, B = 0, C = 7, D = -6$

(C) $A = 1, B = 0, C = -7, D = -6$

(D) $A = 1, B = 0, C = 7, D = 6$

Sr. No. 249

ENTRANCE TEST-2016

FACULTY OF PHYSICAL AND MATERIAL SCIENCE

M.A./M.Sc. STATISTICS

Option–I : Statistics

Option–II : Mathematics

Total Questions : **Option–I : 60**
Option–II : 60

Time Allowed : **70 Minutes**

Question Booklet Series **A**

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be 'Negative Marking' for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain Positive Score in Entrance Test Examination shall be eligible for administration.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

SEAL

1. Geometric mean of three numbers 2, 4, 8 is :

   (A) 4.67 (B) 4

   (C) 5 (D) None of these

2. Arithmetic mean of the first n natural numbers is :

   (A) $(n+1)/2$ (B) $n/2$

   (C) $n(n+1)/2$ (D) None of these

3. For describing the variation in the data, we prefer which of the following measures ?

   (A) Mean Deviation (B) Range

   (C) Standard Deviation (D) None of these

4. If mean, median and mode of a distribution are same, then the distribution is called :

   (A) Leptokurtic (B) Mesokurtic

   (C) Platykurtic (D) None of these

5. If correlation coefficient between two variables is zero then :

   (A) Two variables are independent

   (B) Two variables are uncorrelated

   (C) No relationship between two variables

   (D) None of these

6. If b_{xy} and b_{yx} are 4/5 and 9/20 then r_{xy} is :

   (A) -0.6 (B) .76

   (C) .8 (D) 0.6

7. The best regression line for the following data :

Age (in months) :	1	2	3	3	4	5
Weight (in Kg) :	3	4	5	6	7	8

   is :

   (A) $Y = 2 + Age$ (B) $X = 2 + Weight$

   (C) $Y = 1 + 2Age$ (D) None of these

8. For higher degree of correlation between the two variables, the angle between the lines will be :

(A) Higher
(B) Smaller
(C) Constant
(D) None of these

9. The probability of getting 2 or 5 when a dic is tossed :

(A) 1/3
(B) 1/4
(C) 1/5
(D) None of these

10. The probability of selecting a black card or a 6 from a deck of 52 cards equals :

(A) 15/32
(B) 4/52
(C) 7/13
(D) None of these

11. In a class, 40% of the students study Mathematics and Science. 60% of the students study Mathematics. Then the probability of a student studying Science given that he/she is already studying Mathematics is :

(A) 1/2
(B) 2/3
(C) 5/3
(D) None of these

12. Probability of an event A lies between :

(A) 0 and 1
(B) –1 and 1
(C) 0 or 1
(D) None of these

13. Given that $E[X+4] = 10$ and $E[X+4]^2 = 116$, then $E[x^2]$ is equal to :

(A) 24
(B) 52
(C) 36
(D) None of these

14. If Mean of a Poisson distribution is 4, then the standard deviation is :

(A) 5
(B) 4
(C) 2
(D) None of these

15. $M_x = e^{\lambda(e^t - 1)}$ is the moment generating function of which of the following distributions ?

(A) Normal
(B) Poisson
(C) Binomial
(D) Exponential

16. Variance of geometric distribution is :

(A) pq (B) p/q

(C) npq (D) q/p^2

17. If moment generating function of a distribution is $e^{5t+\frac{1}{2}t^2}$ then variance of the distribution is :

(A) 1 (B) 2

(C) 5 (D) None of these

18. The probability that a normal variate will lie within the range $\mu \pm 3\sigma$ is :

(A) .9997 (B) .0027

(C) .9993 (D) .001

19. If X is uniformly distributed with mean 1 and variance 4/3, then $P(X<0)$ is :

(A) 1/2 (B) 1/3

(C) 1/4 (D) 1/5

20. If X follows exponential distribution with parameter θ, then variance will be equal to mean if :

(A) $\theta > 1$ (B) $\theta = 0$

(C) $\theta < 1$ (D) $\theta = 1$

21. Sample size is treated as small if it is :

(A) > 40 (B) > 30

(C) ≤ 30 (D) None of these

22. Which of the following Errors is treated as more serious ?

(A) Type I (B) Type II

(C) Level of significance (D) None of these

23. If a hypothesis H_0 is rejected at .05 level of significance, then it :

(A) Will be accepted at .01 level of significance

(B) Will be rejected at .01 level of significance

(C) May be rejected at .01 level of significance

(D) None of these

24. For testing sample mean versus population mean of 10 randomly selected samples, we use which of the following test statistics ?

(A) $\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ (B) $\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$

(C) $\frac{\bar{x}-\mu}{\frac{s^2}{\sqrt{n}}}$ (D) $\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

25. If X follows normal distribution then $Z^2 = \frac{(x-\mu)^2}{\sigma}$ is a Chi-Square variate with :

(A) n d.f. (B) n–1 d.f.

(C) 1 d.f. (D) None of these

26. In a paired t-test, we are given that $\bar{d} = 2.58, S = 3.09, n = 12$ and $t_{0.05}$ for 11 d.f. is 2.89 then H_0 :

(A) May be rejected (B) Will be accepted

(C) May be accepted (D) Will be rejected

27. A random sample of 27 pairs of observations from a normal population gave $r = 0.6$. If $t_{0.05}$ for 25 d.f. = 2.06, then r is :

(A) Significant (B) In-significant

(C) Least significant (D) None of these

28. For testing equality of two population variances, we use :

(A) Paired t-test (B) Z-test

(C) T-test (D) F-test

29. Census is conducted in India every :

(A) 5 years (B) 10 years

(C) 15 years (D) None of these

30. _____ is a set of elements taken from a larger population according to certain rules.

(A) Samples (B) Statistic

(C) Population (D) None of these

31. If each member of a population has an equal chance of being selected, then this is called:

(A) A non-random Sampling
(B) Quota Sampling
(C) A Snowball Sampling
(D) None of these

32. Which of the following is not an advantage of Sampling over census ?

(A) Less Time
(B) Less Cost
(C) More Precise Results
(D) None of these

33. If 5 samples out of a population having 100 units are to be selected using Systematic Sampling, then every _____ population unit is to be selected after the first sample is randomly selected.

(A) 5^{th}
(B) 10^{th}
(C) 15^{th}
(D) 20^{th}

34. In order to study socio-economic conditions of university employees, we should use :

(A) Simple Random Sampling
(B) Stratified Sampling
(C) Cluster Sampling
(D) None of these

35. If the population consists of a linear trend, then which of the following inequalities is true ?

(A) $Var(\bar{y}_{st}) \leq Var(\bar{y}_{sys}) \leq Var(\bar{y}_{n})$ R
(B) $Var(\bar{y}_{st}) \leq Var(\bar{y}_{sys}) \geq Var(\bar{y}_{n})$R
(C) $Var(\bar{y}_{st}) \geq Var(\bar{y}_{sys}) \geq Var(\bar{y}_{n})$R
(D) None of these

36. If the sample size increases then standard error :

(A) Increases
(B) Decreases
(C) Does not change
(D) None of these

37. In a CRD with 4 treatments replicated 5 times, then error d.f. will be :

(A) 19 (B) 3

(C) 16 (D) None of these

38. RBD is used where we have :

(A) One way variation (B) Two way variation

(C) Three way variation (D) None of these

39. The basic principles of the experimental designs are :

(A) Blocks, Experimental Units, Treatments

(B) Replication, Randomization, Local Control

(C) Replication, Blocks, Treatments

(D) Blocks, Experimental Units, Randomization

40. Mean Sum of Squares due to treatments for RBD with k treatments and r blocks is calculated as :

(A) SSB/(k – 1) (B) SST/(r – 1)

(C) SST/(k – 1) (D) None of these

41. Consider the following LPP : Maximize $Z = 50x + 18y$

Sub. to $2x + y \leq 100$, $x + y \leq 80$, $x, y \geq 0$,

then which of the following is an optimal solution to the above problem ?

(A) (40, 21) (B) (20, 40)

(C) (20, 60) (D) (50, 0)

42. A solution of an LPP is said to be basic if it satisfies :

(A) Set of constraints (B) Objective function

(C) Non-negativity Condition (D) None of these

43. If any of the Zj-Cj in the final optimal table of Simplex for non basic variable is at zero level, then it is an indication of :

(A) Infeasibility (B) Feasibility

(C) Optimal solution (D) Alternative optimal solution

Consider the following Transportation Problem and answer Q. No. 44-45 :

Destinations

Origins	1	2	3	4	Supply
1	3	1	7	4	300
2	2	6	5	9	400
3	8	3	3	2	500
Demand	250	350	400	200	1200

44. The feasible solution to the above T.P. by North-West Corner Rule is :

(A) 3000
(B) 2500
(C) 4400
(D) None of these

45. The feasible solution to the above T.P. must contain ____ basic cells (allocations).

(A) 5
(B) 6
(C) 7
(D) 8

46. The geometric mean of Laspeyre's and Paasche's indices is :

(A) Bowley's ideal Index
(B) Marshal and Edgeworth's Index
(C) Chain Index
(D) Fisher's Index

47. Seasonal variations repeat with a cycle of :

(A) Five years
(B) Seven years
(C) Two years
(D) A year

48. The Time Reversal Test is satisfied if :

(A) $P_{0n} \times P_{n0} = 1$
(B) $P_{0n} \times Q_{0n} = \frac{\sum p_n q_n}{\sum p_0 q_0}$
(C) $P_{0n} \times P_{0n} = 1$
(D) None of these

49. Vital rates are customarily expressed as :

(A) per ten thousand
(B) percentages
(C) per million
(D) per thousand

50. Crude Birth rate is calculated as :

(A) (Total No. of Live Births during a Year/Total Mean Population during the same year) × 1000

(B) (Total No. of Live Births during a Year/Total Mean Population during the same year) × 100

(C) (Total No. of deaths during a Year/Total Population during the same year) × 100

(D) None of these

51. Life table is also known as :

(A) Survival Table

(B) Life Expectancy Table

(C) Mortality Table

(D) None of these

52. Sex Ratio of Jammu and Kashmir as per 2011 Census is :

(A) 889

(B) 817

(C) 945

(D) None of these

53. Life expectancy in India is about :

(A) 50 years

(B) 58 years

(C) 68 years

(D) None of these

54. The process does not meet the specifications if :

(A) $6\sigma < U - L$

(B) $6\sigma = U - L$

(C) $6\sigma > U - L$

(D) $6\sigma = \frac{U + L}{2}$

where U and L are upper and lower specification limits respectively.

55. For an $\overline{X}$ - chart, when the process is under control, the average run length (ARL) is equal to :

(A) $\frac{1}{\alpha}$

(B) $\frac{1}{1 - \alpha}$

(C) $\frac{1}{\beta}$

(D) $\frac{1}{1 - \beta}$

56. The $\overline{X}$ chart is more effective to detect shifts in process mean if the magnitude of shifts is:

(A) Greater than 1.5σ (B) Lesser than 1.5σ

(C) Equal to 1.5σ (D) None of these

57. Let $x_1, x_2, ..., x_n \sim N(\mu, 1)$ then $T = \frac{\sum xi^2}{n}$ is an unbiased estimator of:

(A) μ^2 (B) $\mu^2 + 2$

(C) $\mu^2 - 1$ (D) $\mu^2 + 1$

58. If the level of confidence increases then the interval containing the estimated value of the unknown parameter will:

(A) Become narrow (B) Become wider

(C) Remain the same (D) None of these

59. Which of the following is not the requirement of a good estimator?

(A) Unbiasedness (B) Consistency

(C) Symmetry (D) Sufficiency

60. A _____ is a numerical characteristic of a sample and a _____ is a numerical characteristic of population.

(A) Sample, Population (B) Population, Sample

(C) Statistic, Parameter (D) Parameter, Statistic

1. Which of the following is true for the function :

$$f(x) = \begin{cases} x^2 & \text{for } x \leq 0 \\ 1 & \text{for } 0 < x < 1 \\ \frac{1}{x} & \text{for } x > 1 \end{cases}$$

(A) Continuous at $x = 0$

(B) Differentiable at $x = 0$

(C) Discontinuous at $x = 0$ and non-differentiable at $x = 0$

(D) None of the above

2. $\lim_{n \to \infty} \frac{3 + 2\sqrt{n}}{\sqrt{n}}$ equals :

(A) 2

(B) $\sqrt{2}$

(C) $\frac{1}{2}$

(D) None of the above

3. Which of the following is true for the function $f(x) = x(x-1)(x-2)$ defined in $\left[0, \frac{1}{2}\right]$?

(A) Lagrange's mean value theorem is not applicable

(B) Lagrange's mean value theorem is applicable with $C = \frac{6 - \sqrt{21}}{6}$

(C) Rolle's theorem is applicable with $C = \frac{6 - \sqrt{21}}{6}$

(D) None of the above

4. Let $f(x)$ and $g(x)$ be two functions. For the Cauchy's mean value theorem to hold, which of the following conditions is not required ?

(A) $f(x)$ and $g(x)$ are both continuous in $[a, b]$

(B) $f'(x)$ and $g'(x)$ both exist in (a, b) and both do not vanish simultaneously

(C) $g(a) \neq g(b)$

(D) $f(a) \neq f(b)$

5. $\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$ equals :

(A) $\frac{1}{3}$ (B) $\frac{-1}{3}$

(C) $\frac{1}{2}$ (D) None of the above

6. If $u = x f\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ equals :

(A) 2u (B) u^2

(C) u (D) None of the above

7. Which of the following is true for the curve $f(x) = e^x$?

(A) Concave downwards throughout (B) Concave upwards

(C) Convex upwards (D) None of the above

8. What is the point of inflexion on the curve $r^2\theta = a^2$?

(A) $\left(\frac{1}{2}, \pm\sqrt{2}\,a\right)$ (B) $\left(\pm\sqrt{2}\,a, \frac{1}{2}\right)$

(C) $\left(1, \pm\sqrt{2}\,a\right)$ (D) None of the above

9. The expression $\tan\left\{i \log\left(\frac{a - ib}{a + ib}\right)\right\}$ reduces to :

(A) $\frac{ab}{a^2 + b^2}$ (B) $\frac{2ab}{a^2 - b^2}$

(C) $\frac{2ab}{a^2 + b^2}$ (D) None of the above

10. If $\alpha + i\beta = \cot^{-1} z$, where $z = x + iy$ and α is a constant, then the locus of z is :

(A) $x^2 + y^2 - x \cot 2\alpha - 1 = 0$ (B) $x^2 + y^2 - 2x \cot \alpha - 1 = 0$

(C) $x^2 + y^2 - 2x \cot 2\alpha + 1 = 0$ (D) $x^2 + y^2 - 2x \cot 2\alpha - 1 = 0$

11. $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma + \tan^{-1}\delta$ equals :

(A) $n\pi - \frac{\pi}{2}$ (B) $n\pi + \frac{\pi}{2}$

(C) $n\pi + \frac{\pi}{2} - 8$ (D) None of the above

12. $\log \tan\left(\frac{\pi}{4} + i\frac{x}{2}\right)$ equals :

(A) $i\tan^{-1}(\sinh x)$ (B) $i\tan(\sinh x)$

(C) $i\tan(\sin x)$ (D) None of the above

13. What is the focus of the parabola $x^2 - 6x - 6y + 6 = 0$?

(A) $(1, 3)$ (B) $(1, -3)$

(C) $(2, 3)$ (D) None of the above

14. If a straight line $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$, then :

(A) $c = am$ (B) $c = a + \frac{a}{m}$

(C) $c = am + \frac{a}{m}$ (D) $c = m + \frac{1}{m}$

15. What is the locus of the mid points of the system of parallel chords to the ellipse ?

(A) Straight line passing through origin (B) Parabola with vertex $(0, 0)$

(C) Ellipse with centre $(0, 0)$ (D) None of the above

16. What is the eccentricity of a rectangular hyperbola ?

(A) 1 (B) 2

(C) $\sqrt{3}$ (D) None of the above

17. What is the pole of $\lambda x + my + nz = p$ with respect to the sphere $x^2 + y^2 + z^2 = a^2$?

(A) $\left(\frac{\lambda}{p}, \frac{m}{p}, \frac{n}{p}\right)$

(B) $\left(\frac{a^2\lambda}{p}, \frac{a^2m}{p}, \frac{a^2n}{p}\right)$

(C) $\left(\frac{a^2}{p}, \frac{b^2}{p}, \frac{c^2}{p}\right)$

(D) None of the above

18. What is the equation of the cone whose generators pass through the point (a, b, g) and have their direction cosines satisfying the relation $a\lambda^2 + bm^2 + cn^2 = 0$.

(A) $a(x-\alpha)^2 + b(y-\beta)^2 + c(z-\gamma)^2 = 0$

(B) $a(x+\alpha)^2 + b(y+\beta)^2 + c(z+\gamma)^2 = 0$

(C) $ax^2 + by^2 + cz^2 = a\alpha + b\beta + c\gamma$

(D) None of the above

19. What is the equation of the cylinder whose generators are parallel to the z-axis and intersect the curve $ax^2 + by^2 = 2z$, $\lambda x + my + nz = p$?

(A) $n(ax^2 + by^2) + 2(\lambda x + my + nz) = p$

(B) $n(ax^2 + by^2) + 2(\lambda x + my) = 2p$

(C) $n(ax^2 + by^2) + 2(\lambda x + my) = 0$

(D) None of the above

20. Which of the following is correct ?

(A) A circular cylinder is the locus of a line which moves such that it is always parallel to a fixed line and is always at a constant distance from it

(B) A circular cylinder is the locus of a line which moves such that it is always perpendicular to a fixed line

(C) A circular cylinder is the locus of a point which moves such that it is always at a fixed distance from a fixed point

(D) None of the above

21. What is the order of the differential equation whose general solution is given by $y = c_1 e^{2x+c_2} + c_3 e^x + c_4 \sin(x + c_5)$?

(A) 5

(B) 2

(C) 4

(D) 3

22. What is the solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ with $y(1)=1$?

(A) $y = x \log x + x$
(B) $y = \log x + x$
(C) $y = x \log x + x^2$
(D) None of the above

23. $\int (x \log x + x \sin x)\, dx$ equals:

(A) $x(\log x - \cos x + 1) + \sin x + c$
(B) $x(\log x + \cos x - 1) + \sin x + c$
(C) $x(\log x + \cos x + 1) + \sin x + c$
(D) $x(\log x - \cos x - 1) + \sin x + c$

24. If $\int \frac{f(x)}{\log \sin x} = \log(\log \sin x) + c$, then $f(x)$ equals:

(A) $\sin x$
(B) $\log \sin x$
(C) $\cot x$
(D) None of the above

25. $\lim_{n \to \infty} \left(\frac{1^p + 2^p + 3^p + \ldots + n^p}{n^{p+1}} \right)$ equals:

(A) $\frac{1}{p+1}$
(B) $\frac{1}{p-1}$
(C) $\frac{1}{p+2}$
(D) None of the above

26. The area between $x^2 + y^2 = \pi^2$ and $y = \sin x$ in the first quadrant is equal to:

(A) $\frac{\pi^3}{4}$
(B) $\frac{\pi^3 - 8}{4}$
(C) $\frac{\pi^3 - 16}{4}$
(D) $\frac{\pi^3 - 8}{2}$

27. What is the area bounded by $xy^2 = 4(2-x)$ and the y-axis?

(A) 2π
(B) 4π
(C) 6π
(D) 12π

28. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants, is of :

(A) first order and second degree

(B) first order and first degree

(C) second order and first degree

(D) second order and second degree

29. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then $I + 2A + 3A^2 + \ldots \infty$ equals :

(I is a unit matrix of order 2)

(A) $\begin{bmatrix} 4 & 1 \\ -4 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 2 \\ -3 & -8 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 2 \\ -8 & -3 \end{bmatrix}$

30. The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$ is :

(A) Idempotent

(B) Orthogonal

(C) Involutory

(D) Nilpotent

31. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 3c_1 & 4c_1 & 5c_1 \\ 3c_2 & 4c_2 & 5c_2 \end{vmatrix}$ equals :

(c_1 and c_2 are any integers)

(A) 2

(B) 1

(C) 0

(D) None of the above

32. Let $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$. For $xy \neq 0, x \neq 0$ and $y \neq 0$ which one of the following is true?

(A) D is divisible by x but not y

(B) D is divisible by both x and y

(C) D is divisible by y but not x

(D) D is not divisible by x and not divisible by y

33. If λ is a characteristic root of an orthogonal matrix, then which of the following is true?

(A) $\frac{\lambda}{2}$ is not a characteristic root (B) $\frac{1}{\lambda}$ is also a characteristic root

(C) $-\lambda$ is not a characteristic root (D) None of the above

34. Which of the following is true?

(A) A real orthogonal matrix has no real characteristic roots other than ± 1

(B) A real orthogonal matrix has real characteristic roots other than ± 1

(C) The characteristic roots of a real orthogonal matrix are of modulus 2

(D) None of the above

35. Which of the following is not true?

(A) The product of two orthogonal matrices of the same order is orthogonal

(B) The inverse of an orthogonal matrix is also orthogonal

(C) The row rank of a matrix is equal to its column rank

(D) The rank of a singular matrix of order n is always n

36. Let A be any matrix and let $B = AA^T$ (where A^T is the transpose of A). Which of the following is true?

(A) B is Hermitian (B) B is skew-symmetric

(C) B is symmetric (D) None of the above

37. If one root of the equation $x^4+2x^3-16x^2-22x+7=0$ is $\alpha = 2+\sqrt{3}$, and the other roots are β, γ and δ. Then $\alpha^2+\beta^2+\gamma^2+\delta^2$ equals :

(A) 36 (B) $36+12\sqrt{2}$

(C) $36-12\sqrt{2}$ (D) None of the above

38. Let the roots of the equation $6x^3-11x^2+6x-1=0$ be in harmonic progression. If the roots are α, β and γ then $\frac{1}{\alpha}+\frac{1}{\beta}-\gamma$ equals ($\alpha<1$ and $\beta<1$) :

(A) 2 (B) 4

(C) 6 (D) None of the above

39. If α, β, and γ are the roots of the equation $x^3-px^2+qx-r=0$, then $\sum\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$ equals :

(A) $\frac{pq-3}{r}$ (B) $\frac{pq+r}{2}$

(C) $\frac{3pq-r}{r}$ (D) $\frac{pq-3r}{r}$

40. If $\alpha-i\beta$ is a root of the equation $x^3+qx+r=0$ then 2α is the root of the equation :

(A) $x^3+px+r=0$ (B) $x^3-qx+r=0$

(C) $x^3+qx-r=0$ (D) None of the above

41. The sequence $\left\langle\left(1+\frac{1}{n}\right)^n\right\rangle$ is :

(A) Convergent (B) Divergent

(C) Oscillates (D) None of the above

42. The sequence $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+.....+\frac{1}{\sqrt{n}}$ is :

(A) Convergent (B) Divergent

(C) Neither convergent nor divergent (D) None of the above

43. The function f(x) on R defined by :

$$f(x) = \begin{cases} 1, & \text{when x is rational} \\ -1, & \text{when x is irrational} \end{cases}$$

Which one of the following is true for f(x) ?

(A) It is not Dirichlet's function
(B) It is continuous for every $x \in R$
(C) It is discontinuous for every $x \in R$
(D) None of the above

44. If f(x) is continuous in [a, b] and f(a) and f(b) have opposite signs, then there is at least one value of x for which f(x) is :

(A) 1
(B) –1
(C) e
(D) 0

45. If f(z) is analytic inside and on a simple curve C and is not identically equal to a constant, then :

(A) minimum value of | f(z) | occurs on C
(B) maximum value of | f(z) | occurs on C
(C) | f(z) | < M for some constant
(D) none of the above

46. What is the value of the integral $\int_C \frac{1}{z-a} dz$ round a circle whose equation is $|z-a| = \rho$?

(A) 2π
(B) π
(C) $2\pi i$
(D) None of the above

47. If C is the square with vertices at $\pm 2 \pm 2i$, then $\int_C \frac{\cosh z}{z^3} dz$ equals :

(A) $i\pi$
(B) π
(C) $\frac{\pi}{2}$
(D) $\frac{i\pi}{2}$

48. $\int_0^{\infty} \frac{dx}{x^4+1}$ equals :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$ (D) None of the above

49. If $f(x,y)=\begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & \text{otherwise} \end{cases}$

then $\lim_{(x,y)\to(0,0)} f(x,y)$ equals :

(A) 1 (B) 2

(C) 0 (D) does not exist

50. The function $f(x,y)=\begin{cases} \frac{x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$ is :

(A) continuous at (0, 0) (B) discontinuous at (0, 0)

(C) continuous at (1, 1) (D) differentiable at (0, 0)

51. Which one of the following is true for the function ?

$$f(x,y)=\begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

(A) not differentiable at (0, 0)

(B) differentiable at (0, 0)

(C) continuous and differentiable at (0, 0)

(D) none of the above

52. Which one of the following is true for the function $f(x,y)=x^2-3xy^2+2y^4$?

(A) has maximum at (0, 0)

(B) has minimum at (0, 0)

(C) has neither maximum nor minimum at (0, 0)

(D) none of the above

53. Which one of the following is the generator of the cyclic group $G = \{1, -1, i, -i\}$ under multiplication?

(A) -1 (B) 1

(C) $-i$ (D) None of the above

54. Let H and K be two subgroups of a group G. Then HK is a subgroup of G if and only if :

(A) $HK \neq KH$ (B) $H = K^{-1}$

(C) $K = H^{-1}$ (D) $HK = KH$

55. The order of a subgroup H of a finite group G is the divisor of the order of the group G is:

(A) Cauchy's theorem (B) Lagrange's theorem

(C) Euler's theorem (D) None of the above

56. Let a be an element of order n in a group G and let p be a prime number. What is the order of a^p in G ?

(A) n (B) p

(C) $n - p$ (D) none of the above

57. Which of the following is true ?

(A) In a field, the unity and zero are not distinct elements

(B) Every field is an integral domain

(C) The multiplicative inverse of a non-zero element of a field is not unique

(D) None of the above

58. Which of the following is not true ?

(A) A finite commutative ring without zero divisors is a field

(B) The isomorphic image of a ring R without zero divisors is a ring without zero divisors

(C) Isomorphic image of a division ring is a division ring

(D) Isomorphic image of an integral domain is not an integral domain

59. A subset W of a vector space V(F) is a subspace of V if and only if:

(A) $a, b \in F; \alpha, \beta \in V \Rightarrow \alpha - \beta \in W$

(B) $a, b \in F; \alpha, \beta \in V \Rightarrow a + b \in W$

(C) $a, b \in F; \alpha, \beta \in V \Rightarrow a\alpha + b\beta \in W$

(D) None of the above

60. Which of the following is not true?

(A) If V(F) is a vector space, then $(-1)\alpha \neq -\alpha$, $\beta + (\alpha - \beta) \neq \alpha$

(B) The intersection of any family of subspaces of a vector space is a subspace

(C) The union of two subspaces W_1 and W_2 is a subspace if and only if one is contained in the other

(D) None of the above

Rough Work